

29 Week 10. Portfolio theory—Overheads

1. Outline

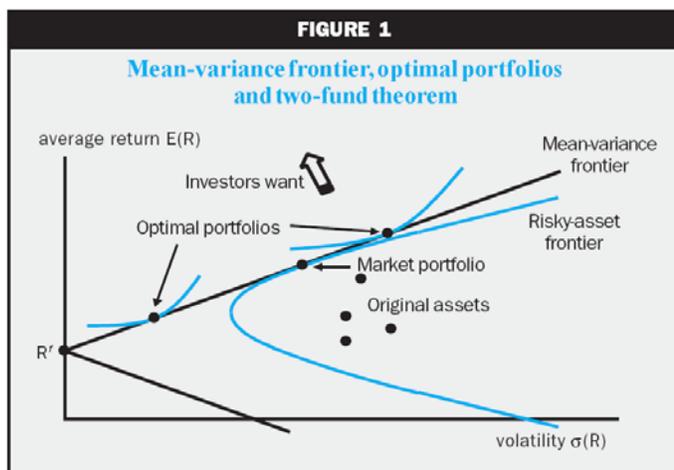
- (a) Mean-variance
- (b) Multifactor portfolios (value etc.)
- (c) Outside income, labor income.
- (d) Taking advantage of predictability.
- (e) Options
- (f) Doubts and opportunities.
- (g) (Contingent claims approach)
- (h) (Taming wild weights, Bayesian portfolio theory)

2. *The average investor holds the market portfolio.*

- (a) Anything other than index is a zero-sum game (relative to the index)
- (b) We can't all rebalance
- (c) How are you different from average? Smarter? Risk? (write/buy insurance)

29.1 Mean-variance – classic results

1. Mean variance reminder



- (a) “Two fund theorem.” *The composition of the stock (tangency) portfolio is the same for all investors.* We’re all out of business.
- (b) No tailored portfolios.
- (c) Think of the *portfolio*, not assets in isolation.
- (d) Names and styles don’t matter. Only means, betas, and covariances matter.

(e) “Passive,” then optimally-diversified “active”.

2. *Portfolio maximization problems.* Where did this come from?

(a) The simplest example:

$$\begin{aligned} & \max_{[w]} E[u(c_{t+1})] \\ c_{t+1} &= W_{t+1} = R_{t+1}^p W_t \\ R_{t+1}^p &= (1-w)R_t^f + wR_{t+1} = R_t^f + wR_{t+1}^e \\ & \max_{[w]} E \left\{ u \left[\left(R_t^f + wR_{t+1}^e \right) W_0 \right] \right\} \\ \frac{d}{dw} : E \left\{ u' \left[\left(R_t^f + wR_{t+1}^e \right) W_0 \right] R_{t+1}^e \right\} &= 0 \end{aligned}$$

Solve for w . (Problem set)

(b) Hey wait a minute, this is

$$E \{ u' (c_{t+1}) R_{t+1}^e \} = 0,$$

the same $p = E(mx)$ equation we’ve been looking at along!

(c) In our theory section, we fixed the properties of *consumption* and saw how returns would have to adjust. Here we fix the properties (mean, variance) of *returns*, and see how *consumption* (i.e. the portfolio weight w which governs consumption) adjusts. Resolution: previously “what is the *question* (properties of returns) so the answer is ‘hold the market portfolio?’”

(d) Optimal salads vs optimal portfolios.

29.2 The real portfolio problem

Here is the kind of problem we really want to solve.

$$\max_{\{c_t, w_t\}} E \sum_{j=0}^{\infty} \delta^j u(c_t) \text{ or } E \int_0^{\infty} e^{-\rho t} u(c_t) dt;$$

$$\begin{aligned} W_{t+1} &= R_{t+1}^p (W_t + y_t - c_t); \\ R_{t+1}^p &= R_t^f + w_t' R_{t+1}^e \end{aligned}$$

$$\begin{aligned} w_t &= N \times 1 \text{ portfolio weights} \\ R_{t+1}^e &= N \times 1 \text{ asset excess returns} \end{aligned}$$

$$\begin{aligned} E_t(R_{t+1}^e) &= \mu_t, \\ cov_t(R_{t+1}^e, R_{t+1}^{e'}) &= \Sigma_t, \end{aligned}$$

29.3 Mean variance portfolios

1. A special case, easy to solve, easy to interpret.
2. Key result: *If the investor has no job $y_t = 0$, if returns are independent over time ($E_t(R_{t+1}^e) = \text{const}$), and with power utility and lognormal returns, the investor wants to hold a mean-variance efficient portfolio.*
3. **Portfolio formula** (weights):

$$w = \frac{1}{\gamma} \Sigma^{-1} E(R^e)$$

where $\Sigma = \text{cov}(R^e R^{e'})$; $\gamma = \text{investor risk aversion}$

- (a) Composition: $\Sigma^{-1} E(R^e)$. “Optimal diversification” “Tangency portfolio” Formula used in all portfolio optimizing programs!
- (b) Scale: $1/\gamma$. “Two-fund theorem.”
- (c) Optimal diversification: Σ takes care of the correlation of assets.
- (d) “Mean-variance.” This is also the solution to

$$\min \text{var}(R^p) \text{ s.t. } E(R^p) = \mu$$

$$\min_{\{w\}} w' \Sigma w \text{ s.t. } w' E = \mu$$

$$\Sigma w = \lambda E, \rightarrow w = \lambda \Sigma^{-1} E$$

- (e) Numbers: For a single risky asset (“stocks/bonds”)

$$w = \frac{1}{\gamma} \frac{E(R) - R^f}{\sigma^2(R)}$$

$$0.6 = \frac{1}{3.08} \frac{0.06}{0.18^2}$$

4. Portfolio relative to the market:

- (a) *If the investor lives in an economy where everyone else is the same except risk aversion, investors i 's optimal portfolio is*

$$w^i = \frac{\gamma^m}{\gamma^i} w^m$$

with return

$$R^{pi} = R^f + \frac{\gamma^m}{\gamma^i} R^{em}$$

where w^m and R^{em} are the market portfolio, which is on the mean-variance frontier, and γ^m is average risk aversion

- (b) Useful: “what’s your γ ?” vs. “Are you more or less risk averse than average?”
- (c) *If the investor lives in an economy where everyone else is the same except risk aversion, the CAPM holds.*

$$E(R^e) = \beta E(R^{em})$$

(and if the CAPM does not hold, somebody is not following this portfolio advice)

5. **Horizon effects.**

- (a) If returns are lognormal iid and the investor has power utility, the allocation to risky assets is independent of investment horizon.
- (b) Intuition: With iid returns, mean and variance both scale with horizon so

$$w = \frac{1}{\gamma} \frac{E(R)}{\sigma^2(R)}$$

is unaffected.

- (c) The “fallacy of time-diversification” (oft-repeated). The variance of *annualized* returns is lower for longer horizon, but not the variance of *total* returns.

$$\begin{aligned} \sigma^2(r_1 + r_2 + \dots + r_T) &= T\sigma^2(r) \\ \sigma^2\left[\frac{1}{T}(r_1 + r_2 + \dots + r_T)\right] &= \frac{1}{T}\sigma^2(r) \end{aligned}$$

6. **Factor models and alphas**

- (a) If returns follow a factor model, such as the CAPM,

$$R_t^{ei} = \alpha^i + \beta^i R_t^{em} + \varepsilon_t^i$$

State the problem as “portable alpha”—how much w in the market, and how much w_ε to chase α ,

$$R_t^p = R^f + w_m R_t^{em} + w'_\varepsilon (R_t^e - \beta R_t^{em}) = R^f + w_m R_t^{em} + w'_\varepsilon (\alpha + \varepsilon)$$

Answer

$$w_m = \frac{1}{\gamma} \frac{E(R^{em})}{\sigma^2(R^{em})}; \quad w_\varepsilon = \frac{1}{\gamma} \Sigma_\varepsilon^{-1} \alpha$$

Separately think about your factor exposures and your alpha exposures. “Passive” vs. “active”.

- (b) $\Sigma_\varepsilon^{-1} \alpha$ tells you *how much to chase α* . Not infinite! The more you chase α , the less diversified your portfolio.
- (c) Σ_ε^{-1} tells you *how to diversify optimally across your active managers or alpha bets*. This depends on the correlation of their strategies.
- (d) Small alphas with even smaller Σ can be great bets if you can leverage them enough.

7. **What’s wrong**

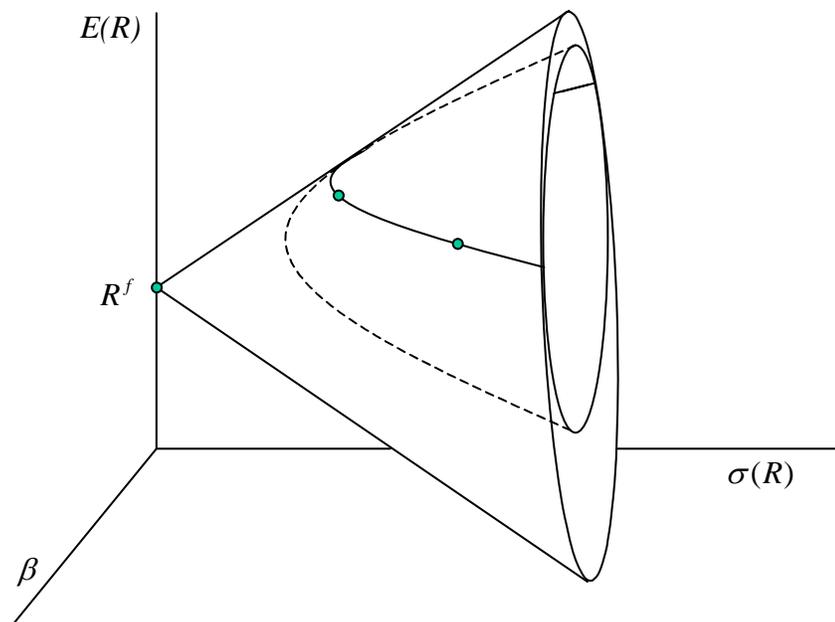
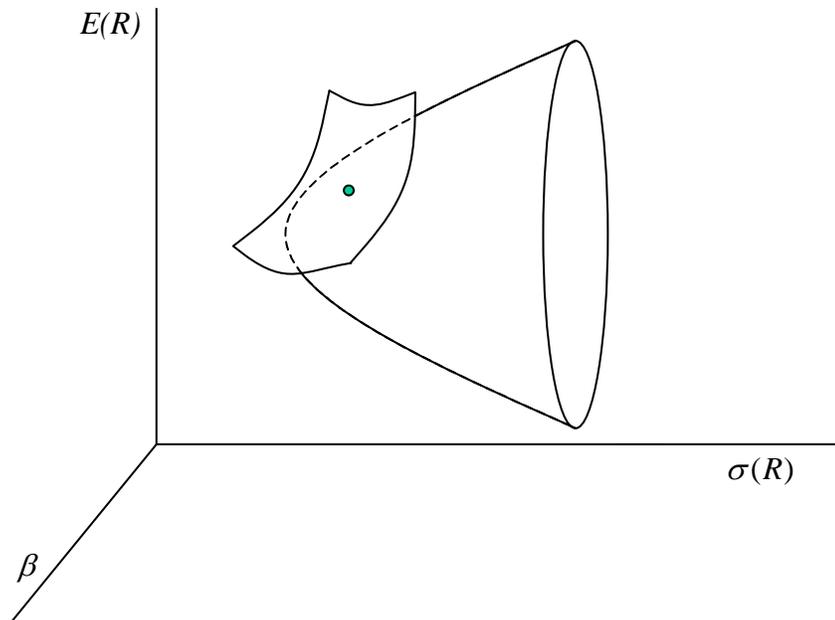
- (a) Assumption 1: No job!
- (b) Assumption 2: Returns are iid, $E_t R_{t+1}$ is constant over time! What about $R_{t+1} = a + b \times DP_t + \varepsilon_{t+1} \rightarrow E_t(R_{t+1}) = a + b \times DP_t$? (Hedge fund use of this formula is particularly silly.)
- (c) Prediction 1: Returns follow the CAPM. If not, somebody is not using this formula. Maybe you shouldn’t either?
- (d) The portfolio problem also says consume proportionally to wealth. $c_t = kW_t \rightarrow \sigma(\Delta \ln c) = \sigma(\Delta \ln W) = 16\%$! If you don’t want to follow this advice, (“consume steadily, the market will bounce back”) why follow the portfolio advice?

29.4 Multifactor/Merton

How do value/growth/momentum, predictability, and the fact that people have jobs or businesses change this picture?

1. Example: A recession factor. Axes: mean $E(R^p)$, standard deviation $\sigma(R^p)$, covariance or β of portfolio with recessions.

(a) MVF becomes a *multifactor efficient frontier*



- (b) The average (and hence market) portfolio *is no longer on the mean-variance frontier*.
- (c) Assumption change: Investors *do* have labor, business income, illiquid portfolio, real estate or other asset that cannot be sold. They want portfolios that “hedge against risks to these state variables.”
- (d) Varying $E_t(R_{t+1})$ works the same way. (“Merton”) Lower $E_t R_{t+1}$ is bad news, so you want assets that go up when $E_t R_{t+1}$ (D/P) decline.

2. Math: We solve the problem *with* job/business y_t and/or *with* varying $E_t(R^e), \sigma_t(R^e)$.

$$\max_{\{c_t, w_t\}} E \sum_{j=0}^{\infty} \delta^j u(c_t) \text{ or } E \int_0^{\infty} e^{-\rho t} u(c_t) dt;$$

$$\begin{aligned} W_{t+1} &= R_{t+1}^p (W_t + y_t - c_t); \\ R_{t+1}^p &= R_t^f + w_t' R_{t+1}^e \end{aligned}$$

$$\begin{aligned} w &= N \times 1 \text{ portfolio weights} \\ R^e &= N \times 1 \text{ asset excess returns} \end{aligned}$$

An example of a model for time-varying returns and time-varying outside income: “state variables” z_t , “shocks to asset returns” ε_{t+1}^r and “shocks to state variables” ε_{t+1}^z

$$\begin{aligned} R_{t+1}^e &= a_r + b_r' z_t + \varepsilon_{t+1}^r \\ \text{cov}_t(R_{t+1}^e) &= \Sigma \end{aligned}$$

$$\begin{aligned} y_{t+1} &= a_y + b_y' z_t + \varepsilon_{t+1}^y \\ z_{t+1} &= a_z + b_z' z_t + \varepsilon_{t+1}^z \end{aligned}$$

3. Result: (weights) investors value mean, want to control variance, and want to control *covariance* with “state variables” (outside income, news of future returns)

$$\begin{aligned} w &= \frac{1}{\gamma} \Sigma_t^{-1} E_t(R_{t+1}^e) + \beta_{R,z'} \frac{\eta}{\gamma} \\ &= \text{market timing/mean-variance} + \text{hedging demand} \end{aligned}$$

η = “aversion to state variable risk”. $\beta_{R,z'}$ = beta of returns on “state variable shocks” We add a) *market timing* and b) *hedging demand* for securities.

4. (Returns) *Relative* to market, and others, if everyone is like this: your portfolio return is

$$\begin{aligned} R_{t+1}^p &= R_t^f + \frac{\gamma^m}{\gamma^i} R_{t+1}^{em} + \frac{1}{\gamma^i} (\eta^{i'} - \eta^{m'}) R_{t+1}^{ez} \\ R_{t+1}^{ez} &\equiv \beta'_{R^e, z'} R_{t+1}^e \end{aligned}$$

R^{ez} : fitted value in a regression of z shocks on R^e = hedge portfolio or “mimicking portfolio” for state variable risk. η^m, γ^m market average values.

5. If everyone is like this, an ICAPM holds

$$E(R^{ej}) = \gamma^m \text{cov}(R^{ej}, R^{em}) - \text{cov}(R^{ej}, z') \eta^m$$

6. Implications of hedging demand:

- (a) If you're like everyone else $\eta^i = \eta^m$ you just hold the market, *despite* the value premium.
- (b) Mean-variance investor ($\eta^i = 0$) ("Writes insurance") and gets better than R^{em} to do it! (R^{em} is no longer mean-variance efficient.)
- (c) Investors differ by risk aversion γ^i and aversion to recession risk η^j . *Three (or more) fund theorem.*
- (d) Nonpriced or $\eta^m \neq 0$ matter too. *Hedging outside income risk is job 1, and does not require any alphas.* Our focus on "anomalies" with $\alpha, \eta^m \neq 0$ is misplaced.

7. More recipes

- (a) *We can split the optimal portfolio in two parts. 1) Hedge as much of the outside income as possible, by shorting a portfolio that looks as much like outside income as possible 2) The rest of the portfolio is on the mean-variance frontier.*
- (b) *Hedge outside income to the extent that you're different from everyone else. Then 2) hold market index*

$$R^i = R^f + \frac{\gamma^m}{\gamma^i} R^{em} + \frac{1}{\gamma^i} (\eta^{it} - \eta^{mt}) R^{ez}$$

- (c) "Style coaching" is a reason for tailored portfolios! With fees!
- (d) This is separate business – separate management companies can set up 1) hedge portfolios 2) passive indices 3) alphas.

8. Summary: Two big sources of hedging demand.

- (a) Hedging outside income – jobs, businesses, human capital etc.
- (b) Hedging "intertemporal opportunities" ...

29.5 An example of why extra state variables / intertemporal thinking is important.

1. Normal times, a 60/40 allocation with 6% equity premium and 18% volatility

$$w = \frac{1}{\gamma} \frac{E(R) - R^f}{\sigma^2(R)}$$

$$0.6 = \frac{1}{3.08} \frac{0.06}{0.18^2}$$

2. Oct 2008, monthly volatility rose to 70%. How should we adjust our portfolio weights?

$$\frac{1}{3.08} \frac{0.06}{0.70^2} = 0.0397 = 4\% \text{ equity}$$

- (a) Ok, you say, as prices fall, expected return rose. How much? With prices down you're probably at 40% not 60% equity, so we only need to find

$$\frac{1}{3.08} \frac{E(R^e)}{0.70^2} = 0.40$$

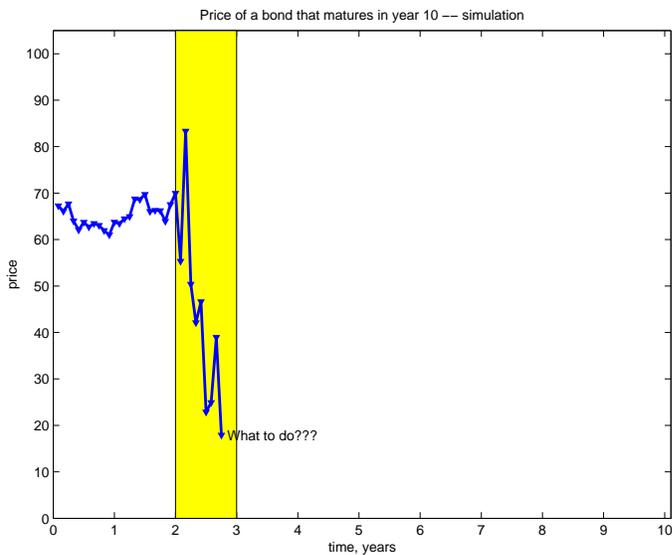
$$E(R^e) = 3.08 \times 0.40 \times 0.70^2 = 0.60 = 60\%$$

- (b) If it makes sense for you to dump, why not everyone else? Formula is not new to science
 (c) Was the market irrationally *optimistic*, and *overvalued*?

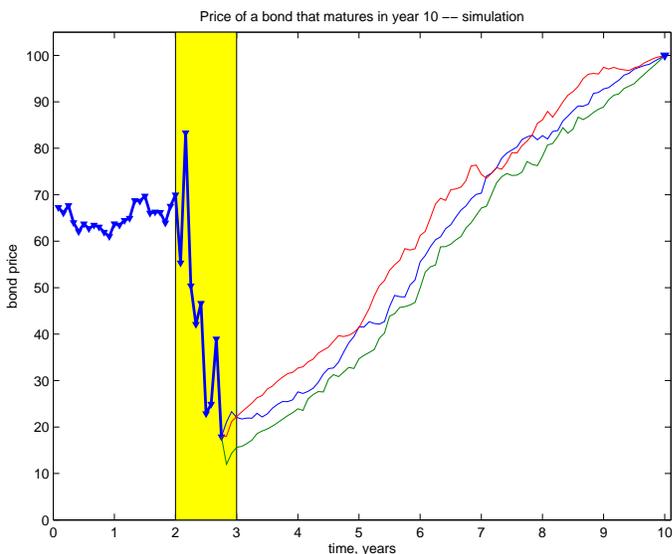
3. Resolution: *This is the wrong formula*. Multifactor/state variable thinking is really important in the real world.

4. To drive the point home, consider long term bond investor story

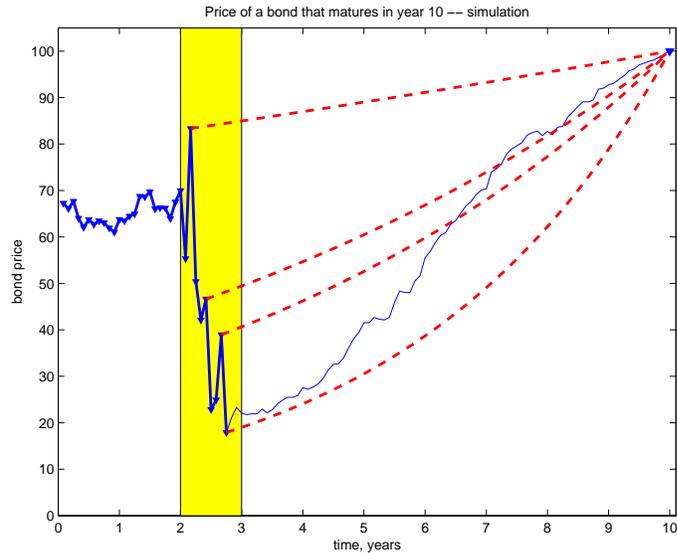
- (a) Liability in 10 years, 10 year zero coupon TIP.



Bonds: Answer: *Do nothing*



Bonds: *Do nothing*



Bonds: *Do nothing*

- (b) For most investors the risk free asset is a coupon-only long-term TIP, not a money market fund, and they should ignore its price fluctuations!
- (c) Viewed Merton-style

$$\text{risky share} = \frac{1}{\text{r.a.}} \frac{\text{expected return} - \text{riskfree rate}}{\text{return variance}}$$

+ “aversion to yield change” \times cov.(return, yield change)

Bonds are a perfect “hedge” against the “state variable”, yield. Cov.(return, yield change) is a strong negative number. The “aversion” η is also a negative number, so you do hold bonds despite $E(R) = R^f$, and huge $\sigma(R)$.

- (d) Long-term bonds also overturn the result that the portfolio is the same for all horizon investors. Can the same apply to stocks...
- (e) Are stocks a bit like bonds, so “do nothing” is at least partially right? Does a price decline mean higher yield (return)? Yes! DP and returns!

5. Bottom line:

- (a) Long term bonds are a *great* example to keep in mind why one-period mean-variance is a terrible guide when there are important “state variables” (changes in ER $\sigma(R)$) *state variable hedge demands really matter*.
- (b) Common practice is to model $E_t(R_{t+1}), \Sigma_t$, and then use one-period maximizers, ignoring hedging demands. This would be a huge mistake in this example, sending you to short term bonds rather than the correct long term bonds.
- (c) *Looking directly at cashflows is much simpler than these one-period formulas! Intertemporal hedging demands (state variables for expected returns) disappear from the long-run portfolio if you look at it right.*

29.6 Predictability

How does stock return forecastability (d/p), bond return forecastability (f-y), exchange rate forecastability ($r^d - r^f$) affect portfolio decisions?

1. Market timing? Horizon effects? Hedge state variable shifts (e.g. volatility)?
2. *Portfolio problem.* Just as above, but use our regressions. For example,

$$\begin{aligned} R_{t+1}^{em} &= a + b(D/P_t) + \varepsilon_{t+1} \\ E_t(R_{t+1}^{em}) &= a + b(D/P_t) \\ (D/P_{t+1}) &= \phi(D/P_t) + v_{t+1} \end{aligned}$$

3. *Answer:*

$$w_t = \frac{1}{\gamma} \Sigma_t^{-1} E_t(R_{t+1}^e) + \frac{\eta}{\gamma} \beta_{R,z',t}; \quad z_t = v_t$$

For one asset,

$$w_t = \frac{1}{\gamma} \frac{E_t(R_{t+1}^e)}{\sigma_t^2(R_{t+1}^e)} + \frac{\eta}{\gamma} \frac{cov(\varepsilon_{t+1}, v_{t+1})}{\sigma_t^2(R_{t+1}^e)}$$

“Market timing” demand plus “hedge demand.”

4. **Hedge demand.** An asset can hedge itself. Example: long term bonds!
 - (a) If a bond yield goes down (bad news) then the bond price goes up (it hedges its own bad news). Hence long term bonds are “safe” for long term investors, despite poor E/σ^2 . ($\eta < 0$, $cov(\cdot) < 0$).
 - (b) Stocks are a bit like bonds – poor returns means lower P/D, higher expected returns – hedging demand! We saw $cov(\varepsilon_{t+1}, v_{t+1})$ is very strong and negative; stocks like bonds are “good hedges for their intertemporal opportunities.”
 - (c) This hedge value increases with horizon, (η is bigger for longer-lived investors) so justifies some horizon effects.
 - (d) Note varying $E_t(R_{t+1})$ (you care) is not enough for hedge demands. You need the covariance (β) as well. (you can do something about it)
 - (e) The market timing demand is what comes out of a one-period optimizer. When you use one-period optimizers in a dynamic environment, you’re ignoring the hedging demand.

5. **Market timing demand:**

- (a) Given a regression model, i.e.

$$R_{t+1}^{em} = a + b \left(\frac{D_t}{P_t} \right) + \varepsilon_{t+1}$$

we have

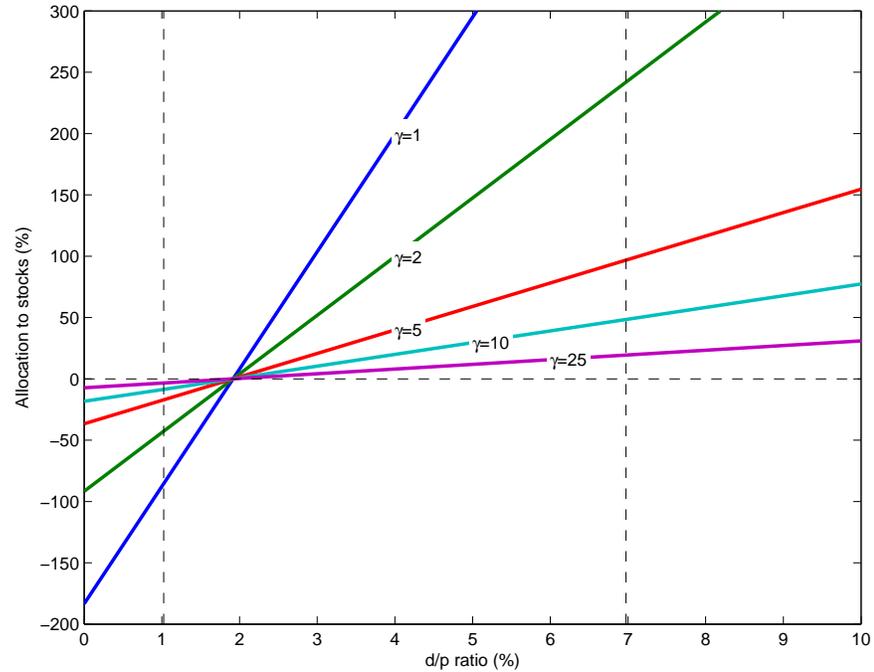
$$E_t(R_{t+1}^{em}) = a + b \left(\frac{D_t}{P_t} \right); \quad \sigma_t^2(R_{t+1}^{em}) = \sigma_\varepsilon^2.$$

We can calculate how much the investor should “market time”

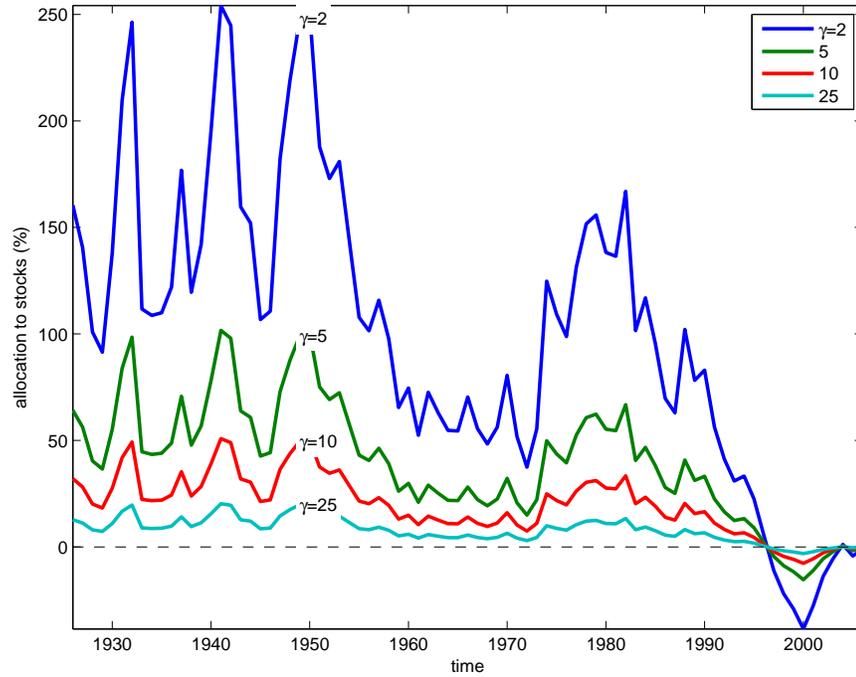
$$w_t = \frac{1}{\gamma} \frac{a + b \left(\frac{D_t}{P_t} \right)}{\sigma_\varepsilon^2}$$

(b) Problem set 1 d/p regression

$$R_{t+1}^e = \begin{matrix} -7.20 \\ (t) \end{matrix} + \begin{matrix} 3.75 \\ (2.66) \end{matrix} D/P_t + \varepsilon_{t+1} \quad \sigma_\varepsilon^2 = 19.81\% \quad (35)$$



Market timing portfolio allocation. The allocation to risky stocks is $\alpha_t = \frac{1}{\gamma} \frac{E_t(R^e)}{\sigma^2}$. Expected excess returns come from the fitted value of a regression of returns on dividend yields, $R_{t+1}^e = a + b(D_t/P_t) + \varepsilon_{t+1}$. The vertical lines mark $E(D/P) \pm 2\sigma(D/P)$.

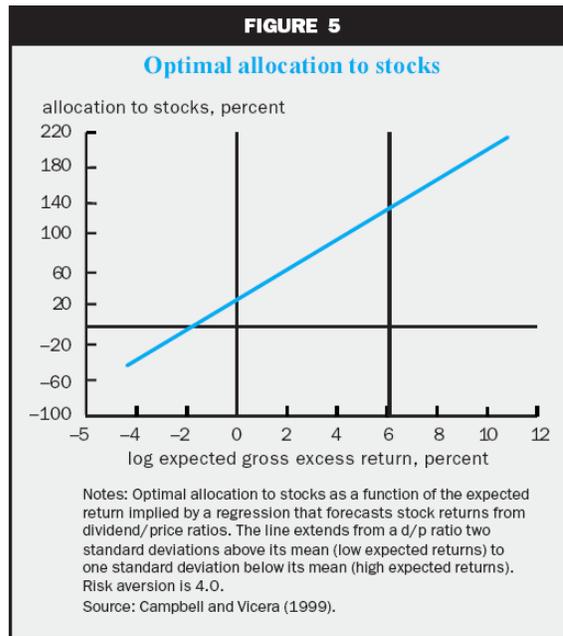


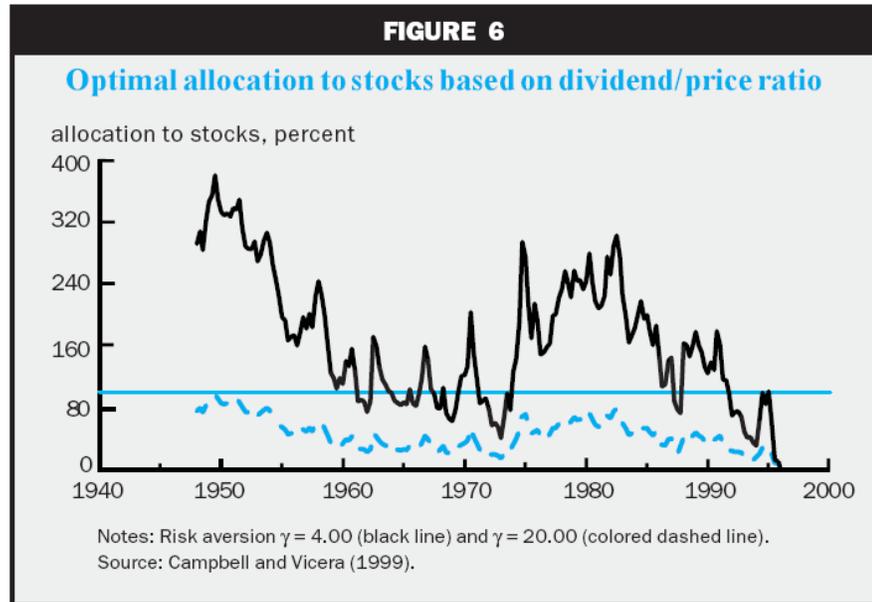
Market timing portfolio allocation over time. The allocation to risky stocks is $\alpha_t = \frac{1}{\gamma} \frac{E_t(R^e)}{\sigma^2}$. Expected excess returns come from the fitted value of a regression of returns on dividend yields,

$$R_{t+1}^e = a + b(D_t/P_t) + \varepsilon_{t+1}.$$

This seems to imply incredibly strong market timing based on d/p!

- (c) A complete calculation by Campbell and Viceira. (Hard math, includes hedge demand)





- (d) *This seems incredibly aggressive! How to tame this wild advice? “Bayesian portfolio theory?”*
- (e) Hedging demands? Harder to calculate.
- (f) Reality? *This is what hedge funds do. Except without hedging demands! And their portfolio optimizers blow up too.*
6. Note: *The average investor holds the market. Why doesn't everyone do this? And if they did, the phenomenon would disappear. Was leaving y out a good idea?*

30 "Mean-Variance Benchmark"

1. The idea: Hedging demands are important. People don't use them. A solution?
2. Example: Long-term bonds. :
 - (a) "Buy long term (indexed, riskfree) bonds because their one-year returns covary negatively with shocks to their reinvestment risks"
 - (b) "Buy long term bonds because at a long horizon, they pay the same amount no matter what happens"

3. Why not look at long horizon portfolios directly? In a very simple example,

$$\max E[u(W_T)] \quad s.t. E(R_T), cov(R_T)$$

- (a) Dynamic strategies generate more R_T like funds.

4. People don't want a single long horizon. They want streams of dividends,

$$\max E \sum_{t=0}^{\infty} \beta^t u(c_t)$$

c_t is the dividend stream from an investment strategy Hence, the paper idea

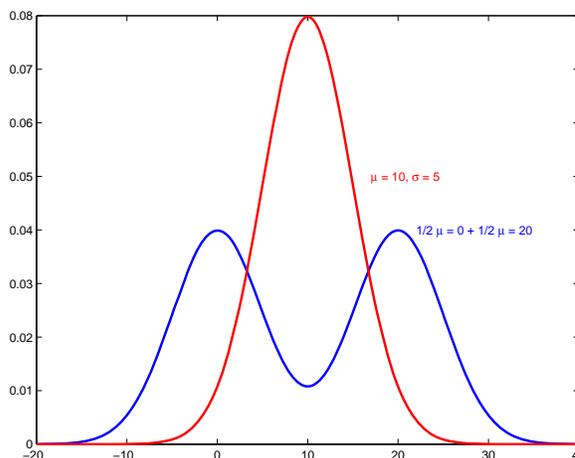
$$E \sum_{t=0}^{\infty} \beta^t u(c_t) = \sum_{t=0}^{\infty} \pi_t \beta^t u(c_t) = \tilde{E}[u(c_t)]$$

If you treat time and probability symmetrically and look at the final payoffs, The long horizon problem is the same as a one-period problem

5. Results: With quadratic utility, even with all sorts of dynamics $E_t(R_{t+1})$ etc.
 - (a) Investors choose $\{c_t\}$ on a "long-run" mean-variance frontier.
 - (b) A "long-run" CAPM .
 - (c) State variables drop out!
 - (d) Outside income does not drop out. Short a portfolio that hedges your outside income, then invest in a long-run mean-variance efficient portfolio
 - (e) A long-run multifactor model with average outside income.
6. Limitations: Quadratic utility
7. The chat. This isn't easy

30.1 Bayesian portfolios, parameter uncertainty, and taming market timing.

1. What about those wacky portfolio weights? So far we have treated models ($\mu, \Sigma, E_t R_{t+1} = a + bDP_t$) as if the parameters were perfectly known, but we don't really know them. Does this matter? Motivation: Think about the DP regression and its wild advice. But $E_t(R_{t+1})$ are not well measured.
2. What's the effect of *parameter uncertainty* on portfolios? How uncertainty adds to your risk.
 - (a) Standard errors are symmetric. They say "you're as likely to be underinvested as overinvested"
 - (b) *Parameter uncertainty is a source of risk to investors, and says to be more conservative.*
 - (c) Example: Suppose $\sigma = 5$. $\mu = 0$ or $\mu = 20$, equally likely. The risk *you face* is not $\mu = E(\mu) = 10, \sigma = 5$.



- (d) How do we incorporate that intuition quantitatively? Bottom line: *Add the standard error of the mean to the variance of the return.*

$$\text{risky share} = \frac{1}{\text{r.a.}} \times \frac{\text{expected return} - \text{riskfree rate}}{\text{return var.} + \text{uncertainty about E return} \Leftarrow$$

$$w = \frac{1}{\gamma} \frac{E(R^e)}{\sigma^2(R^e) + \sigma^2(E(R^e))}$$

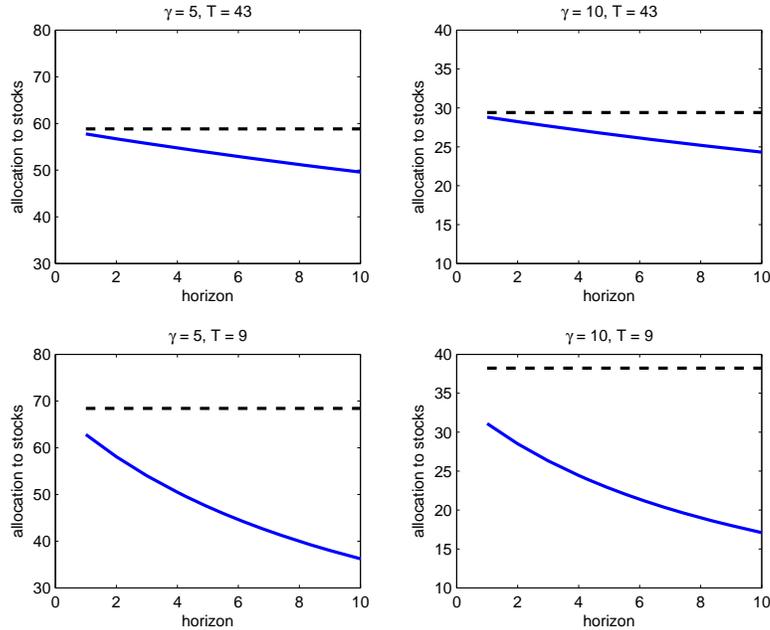
parameter uncertainty makes you take less risk.

- (e) *Important practical application.* The average hedge fund runs a complex model for expected returns, and plugs it in to the optimizer *as if the parameters are known.*

3. Application 1 *Parameter uncertainty makes stocks riskier for long-run investors.*

$$w = \frac{1}{\gamma} \frac{E(R^e)}{\sigma^2(R^e) + \frac{1}{T}\sigma^2(R^e)} = \frac{1}{\gamma} \frac{E(R^e)}{\sigma^2(R^e)(1 + \frac{1}{T})}$$

A calculation, after Barberis (2000).



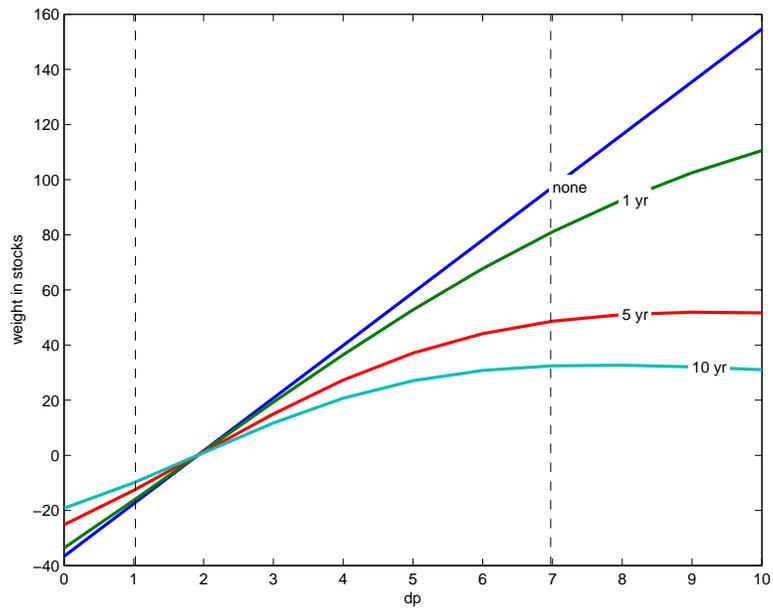
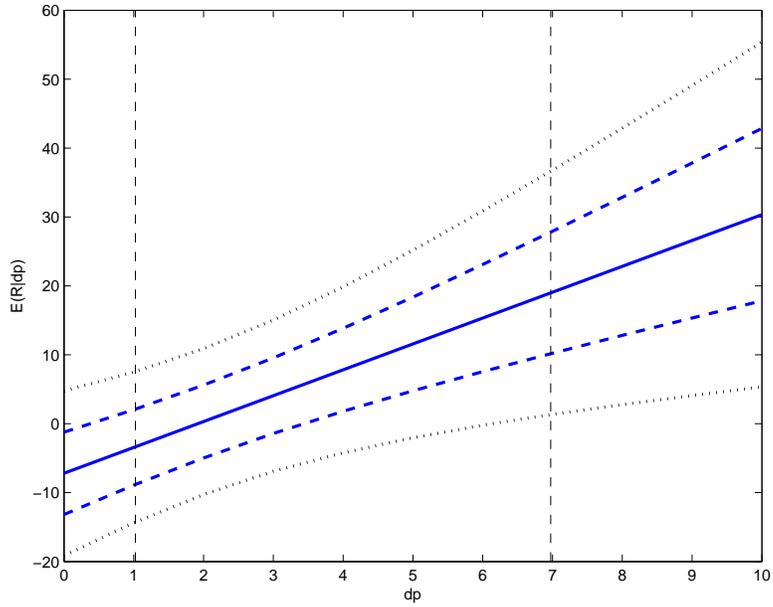
Portfolio allocation to stocks with parameter uncertainty. The solid lines present the case with parameter uncertainty, and the dashed lines ignore parameter uncertainty. The allocation to stocks is $w = \frac{1}{\gamma} \frac{\mu - r}{\sigma^2 \left[1 + \frac{(\gamma - 1)h}{\gamma T} \right]}$. $\mu = 0.06, \sigma = 0.1428$ for $T = 43$ and $\mu = 0.078, \sigma = 0.151$ for $T = 9$ with $r = 0$. (The extra $(\gamma - 1)/\gamma$ allows for continuous rebalancing. See “Portfolio Theory.”)

So, are stocks safer for long term investors?

- (a) Classic result: allocation is independent of horizon (iid returns, known parameters, no outside income)
 - (b) dp, mean reversion, with dp shocks negatively correlated to return shocks : Stocks are like bonds, safer in the long run. (not iid returns)
 - (c) Parameter uncertainty: Stocks are riskier in the long run (parameter uncertainty)
 - (d) And we forgot... the stock/bond nature of wages and outside income.
4. Application to DP regressions: *Recognizing the uncertainty about predictability – variation in $E(R)$ over time leads you to shade portfolios back towards constant weights.*

$$R_{t+1}^e = a + b(D_t/P_t) + \varepsilon_{t+1}$$

$$\sigma^2 [E_t (R_{t+1}^e)] = \sigma^2(\hat{a}) + \sigma^2(\hat{b}) (D_t/P_t)^2 + 2cov(\hat{a}, \hat{b})D_t/P_t$$



$$w = \frac{1}{\gamma \sigma^2(\varepsilon) + \text{horiz} \times \sigma^2 [E_t (R_{t+1}^e)]} \hat{a} + \hat{b}(D_t/P_t)$$

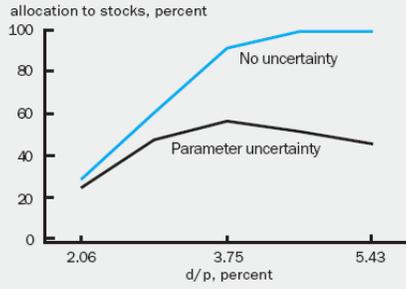
Widening of the error bands is what matters here. *Functional form* uncertainty (squared dp etc) would make it much bigger.

(a) A much more precise (but very hard) calculation by Barberis (2000)

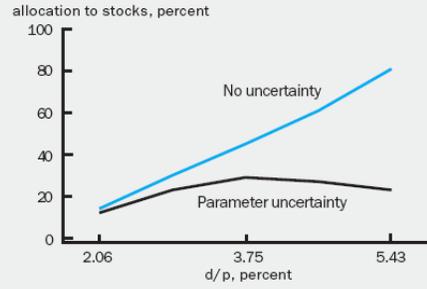
FIGURE 7

Allocation to stocks as a function of dividend/price ratio, with parameter uncertainty

A. Risk aversion coefficient 10



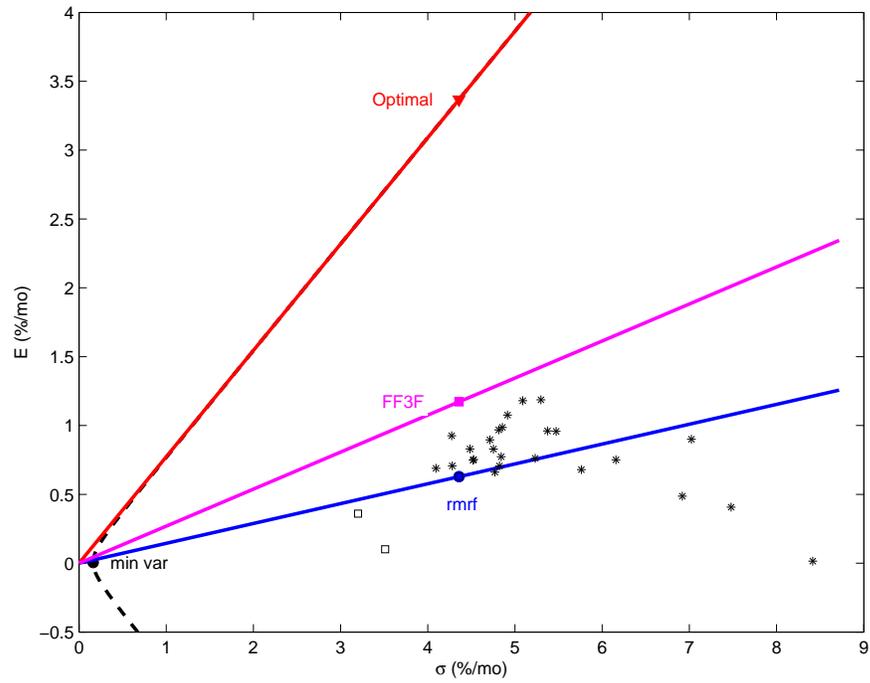
B. Risk aversion coefficient 20



Notes: The colored line ignores parameter uncertainty, as in Campbell and Vicera (1999). The black line includes parameter uncertainty, as in Barberis (1999). Data sample is in months (523).

30.2 Wacky weights in mean-variance analysis.

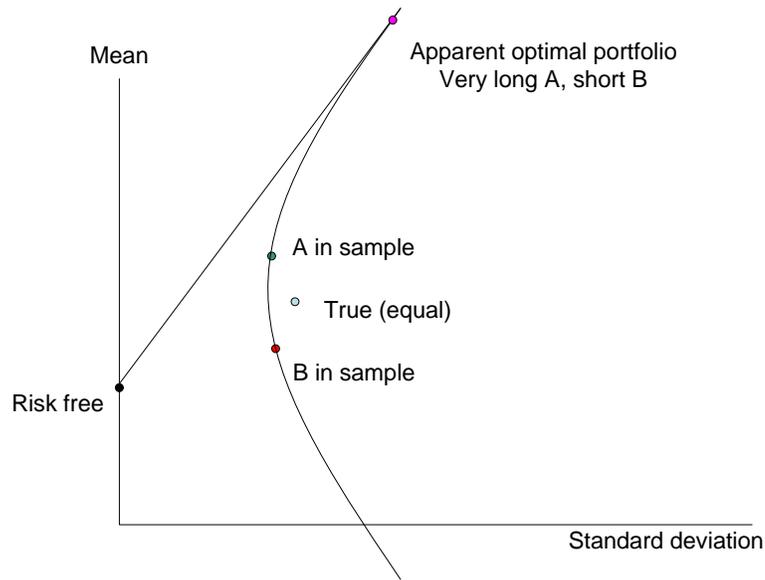
1. Example: FF 25 portfolios + 3 factors. 20 years of data (a lot!)



$$\text{"optimal": } w = \frac{1}{\gamma} \Sigma^{-1} \mu$$

	low	2	3	4	high
small	-149	51	69	96	52
2	-19	-57	190	-13	-60
3	29	-34	-31	-93	41
4	116	-39	-42	35	-2
large	87	-19	8	-22	2
rmrf	hml	smb			
-94	77	-69			

2. Intuition:

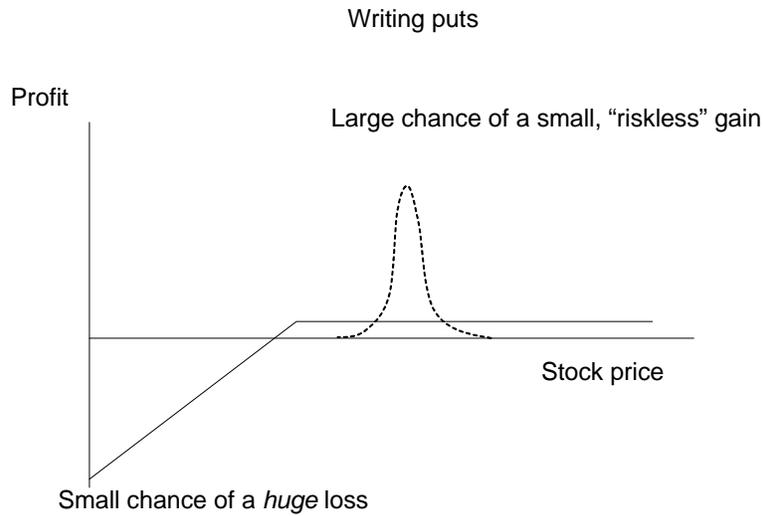


Portfolio optimization problems are very sensitive to mean return inputs, especially when there are highly correlated assets.

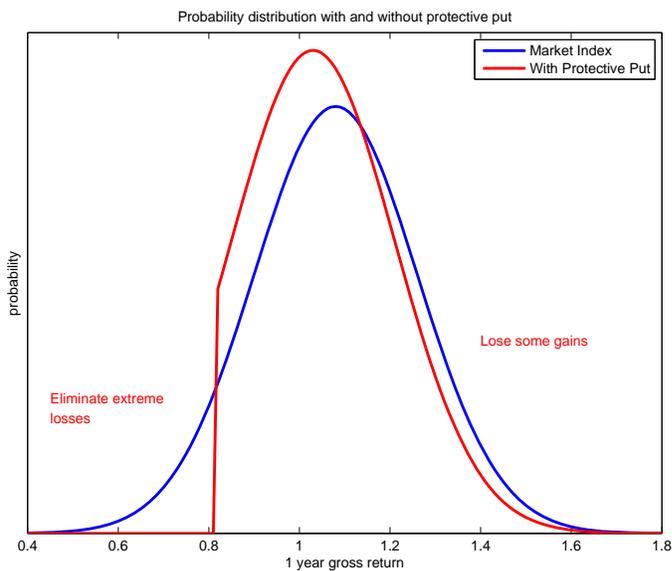
3. What do we do to tame wacky weights?

- (a) Short constraints
- (b) Ad-hoc rules. But miss the opportunities?
- (c) Warning: Standard rules fail when faced with hedge funds/derivatives. 10% exposure in a $\beta = 10$ investment is 100% exposure, so *weights* don't limit *risks*.
- (d) Bayesian approaches? (Optional notes)
- (e) More generally, fit fancy models, treat the parameters as known, feed the results to a black box portfolio optimizer is a very bad strategy! (Read, typical hedge fund.)

30.3 Options



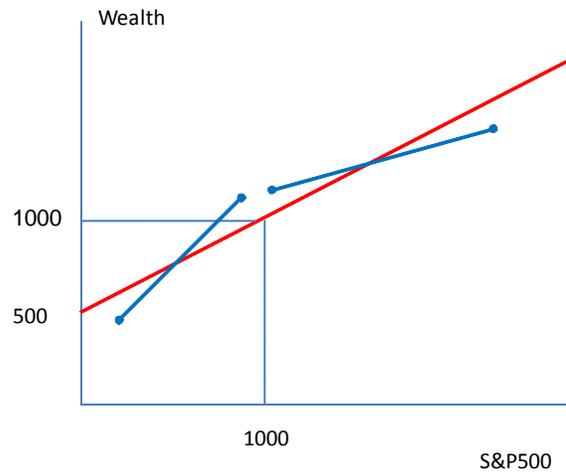
- Write put options? But *the average investor holds the market*. Why are put option premiums so high?
- A: there is a lot of demand to *buy* put options despite the high premium. Very sensibly!
 1. Leverage. “If we lose more than 20%, we default on our debt”
 2. “If we lose more than 20% we have to cut core functions.”
 3. “If we lose more than 20%, our sponsors will give up and fire us.”
 4. Put options are attractive to these investors – despite the cost



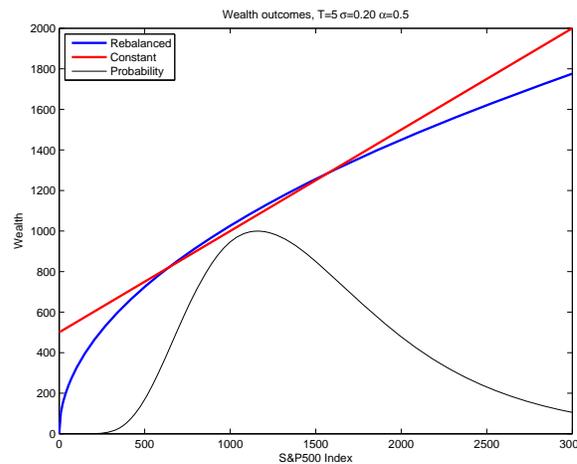
5. *A stop loss order is not the same as a put option*
6. Buy vs. write options? Receive premium or buy insurance despite the large premium?
A: Are you more or less able to take crash risks than the average investor? Really, now?
7. Big picture: You can tailor the entire shape of the return distribution.

- Example 2: Should we rebalance? Wall Street wisdom says yes.

1. *The average investor can't rebalance.*
2. *Rebalancing is just like writing both call and put options.*
3. Example: The S&P500 index is 1,000, you have \$1,000, and you start with 50/50 allocation. If you do not rebalance, your wealth follows the red line. If at S&P=500/1500 you rebalance, you have the blue lines, and the blue dots after two moves. A bet on low volatility. *Just as if you wrote both puts and calls.*



4. *Constantly rebalance? 5 year wealth. A bet on low volatility.*



5. Rebalancing is optimal—if you have power utility returns are iid and lognormal. *They're not. We can't all rebalance!*

30.4 More portfolio theory doubts

1. *The average investor holds the market portfolio.*
 - (a) Predictability: Why do ER vary in the first place?
 - (b) Value, size, momentum: Why is there value in the first place?
 - (c) Why are you different?
2. Catch 22. Anomalies only last if you can't use them.
 - (a) Data-dredging? .
 - (b) Mispricing/overlooked? .
 - (c) Mispricing, but can't arbitrage it due to constraints or transactions costs?
 - (d) Mispricing, but needs institutional reform?
 - (e) Risk factor?
3. Implied *consumption* from portfolio optimization problems is incredibly volatile. If you take the *portfolio advice*, why don't you buy the *consumption advice*? If the model is wrong for consumption why do you believe it for portfolios?
4. Transactions costs!
5. Taxes!

30.5 Good things to say

1. Optimal portfolio 1962: High fee Tailored portfolios.
2. Optimal portfolio 1972: β on market, two fund theorem. No Fee. Maybe chase a little alpha?
3. Optimal portfolio Now:
 - (a) Hedge labor/outside income risks
 - (b) Find desired "priced factor allocation" rrmf, hml, smb, umd, etc.
 - (c) Find right distribution – put options etc.
 - (d) Chase alpha? (alpha=beta you don't understand)
 - (e) Offset implied factor premia in active part.
 - (f) There is a lot for professional advice to offer again!
4. *Investors should hedge their labor or proprietary income risks.* Find important nonpriced factors!
5. Investors do silly things.
6. If you're going to work for a hedge fund, forget all my doubts and learn to do complex optimal portfolios *with good risk management* (covariance matrices).