Problem Set 6 Answers

Here are my results.

1. You can see that means rise to the northeast as for FF, with the same exception for small growth. In this case means seem to be pretty linear with portfolio number, without the S shaped pattern we saw in some other sorts.

```
 Sample
      196301       201012


Time series regression results
As in FF all results are in boxes with size and book to market
mean return
    0.2620     0.7922     0.8295     1.0050     1.1590
    0.4203     0.6967     0.9099     0.9291     1.0272
    0.4354     0.7321     0.7665     0.8648     1.0699
    0.5434     0.5307     0.6905     0.8381     0.8379
    0.4116     0.4637     0.4545     0.5246     0.5895
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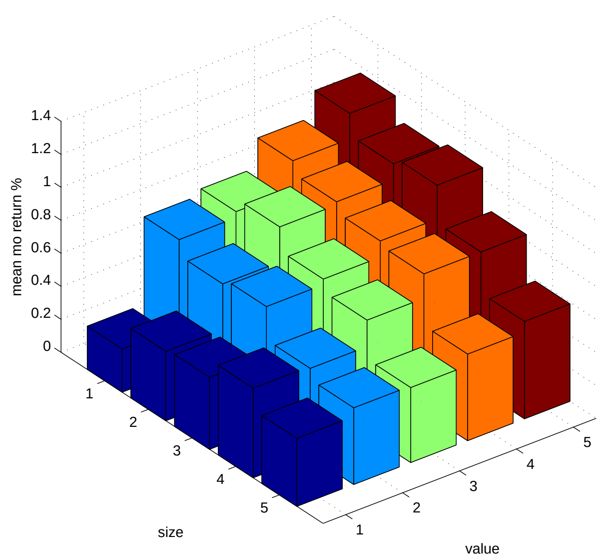


2. a

```
CAPM betas
    1.4349     1.2259     1.0951     1.0154     1.0804
    1.3972     1.1707     1.0490     1.0087     1.1116
    1.3314     1.1133     0.9895     0.9441     1.0202
```

1.2209	1.0809	1.0208	0.9490	1.0362
0.9885	0.9318	0.8730	0.8273	0.8704

CAPM alphas

-0.3952	0.2307	0.3280	0.5400	0.6642
-0.2196	0.1605	0.4295	0.4671	0.5181
-0.1744	0.2223	0.3133	0.4324	0.6026
-0.0158	0.0357	0.2230	0.4034	0.3633
-0.0412	0.0369	0.0546	0.1457	0.1909

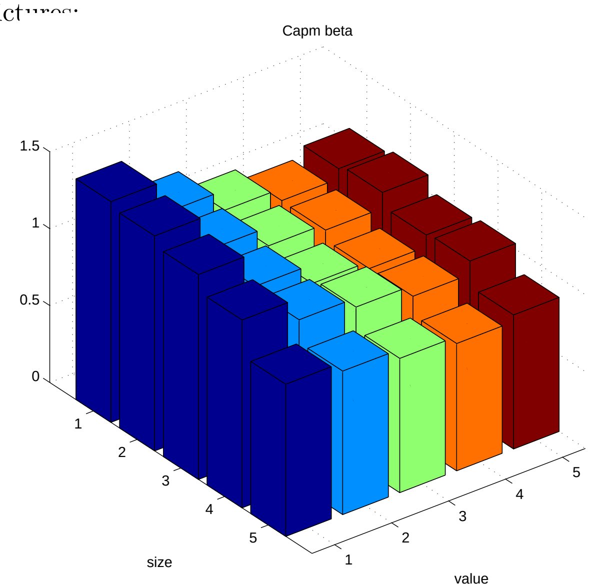
T on CAPM alphas

-1.9361	1.2986	2.2280	3.7547	4.1574
-1.4193	1.2981	3.7161	4.0101	3.6034
-1.3594	2.3201	3.2472	4.1979	4.6619
-0.1625	0.4494	2.4549	4.1954	2.9068
-0.5839	0.5325	0.6496	1.4702	1.4728

CAPM R2

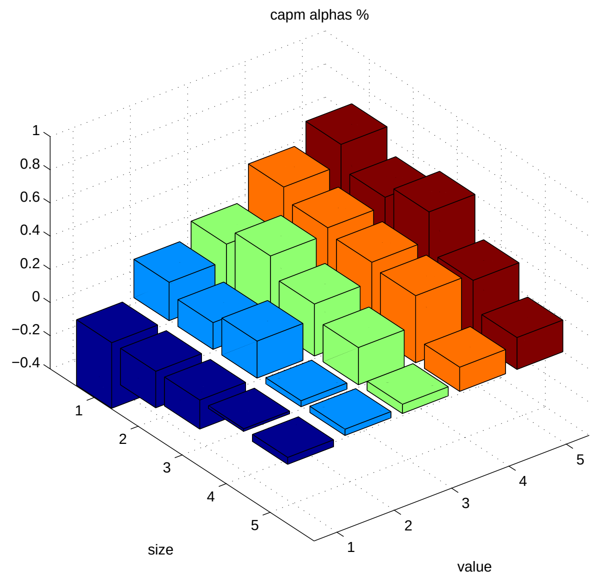
0.6394	0.6309	0.6651	0.6415	0.6214
0.7452	0.7628	0.7472	0.7291	0.6820
0.7944	0.8289	0.7906	0.7509	0.6909
0.8498	0.8693	0.8192	0.7775	0.7115
0.8758	0.8662	0.7945	0.7144	0.6181

The same thing in pictures:



Capm betas *do* vary. They are not all one. The size pattern is ok, they rise to the north. Alas the rise to the north is more pronounced in the growth portfolios where the returns do not rise to the north than it is for the value portfolios where they do. The value pattern is wrong, the betas rise to the northwest not the northeast.

Thus, the alphas are bigger than the mean returns – they are composed of mean returns going one way and betas going the opposite way.



This is exactly how the FF model fails when confronted with momentum by the way.

The CAPM R2 are in the .60–80% range which is typical for large portfolios. There are lots of t stats above 2.

```
CAPM chi2 statistic, N, prob value (%)     108.50    25.00 2.24e-010
10, 5, 1% p values of chi2(N)               34.38    37.65    44.31
F statistic, N, T-N-K, prob value (%)        4.14    25.00   550.00 2.10e-008
10, 5, 1% p values of F(N,T-N-K)             1.39     1.53     1.81
rms alpha, mean abs alpha,                   0.34     0.29
```

Statistically, the CAPM is really rejected. The χ^2 is 108.50 with a 2×10^{-10}(!) probability value. The 10, 5, and 1% cutoff are 34, 38, and 44. How does 108.50 give a 2×10^{-10}% probability value while 44 gives a 1% probability value? The tails of a normal distribution fall quickly. In reality, I bet a bootstrap would give a substantially greater than 2×10^{-10} probability value of a 108.50 χ^2 statistic. The F statistic gives a similar lesson. The rms and mean absolute alphas give a sense of how big they are; about 30 bp (0.30%) per month. This is pretty large compared to the 1% per month (12% per year) level of returns. (You should try to get a sense of what numbers are reasonable.)

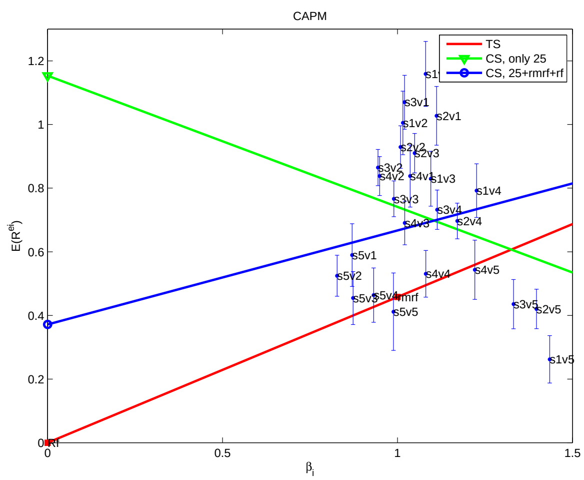
The red line in the picture below gives the *cross-sectional estimate implied by the time-series regression.* When we estimate $R_t^{ei} = \alpha_i + \beta_i R_t^{em} + \varepsilon_t^i$ and look at alphas, what we are really doing is estimating $E(R^{ei}) = \beta E(R^{em}) + \alpha_i -$ we estimate the factor risk premium $\lambda = E(R^{em})$ by running the cross sectional line through the market and risk free rate ignoring all other assets. The alphas are the deviations from this line.

Economically, as you know, the betas go in the wrong direction, so this is a bloody disaster. The alphas are bigger than the spread in average returns

d.

```
comparison table
CAPM, Sample   196301 to   201012
              gamma  lambda s(gamma s(lambd rmse(a)    E|a|    chi2      5%      %p
```

TS		0.46		0.19	0.34	0.29	108.50	37.65	0.00
CS, free g	1.15	-0.41	0.40	0.44	0.23	0.19	68.60	35.17	0.00
CS, with f	0.37	0.30	0.13	0.25	0.26	0.22	108.96	37.65	0.00



i) See table above and the green line. There is a huge intercept and a negative market premium for reasons made clear in the graph. The standard error of $\hat{\lambda}$ is also much bigger. This makes sense; without the information in E(rmrf) and only the slope *across test assets*, with no anchor, there is much more sample variability in $\hat{\lambda}$ estimates. The alphas are smaller of course so the test seems to "reject" much less badly. The key (plot) is that to fit the cross section of *assets* better it gives up on pricing the risk free rate.

Next, see the cross sectional regression that also has assets (rmrf and rf) as test assets. As you can see, now you get much more reasonable results.

Of course GLS says to put all attention on those test assets. Since $R^{em} = 0 + 1 \times R^{em} + 0$, the covariance matrix of errors Σ has a zero in the row and column corresponding to R^{em}, so $E(R^{em})\Sigma^{-1}\beta$ says, put all weight on the market and risk free rate.

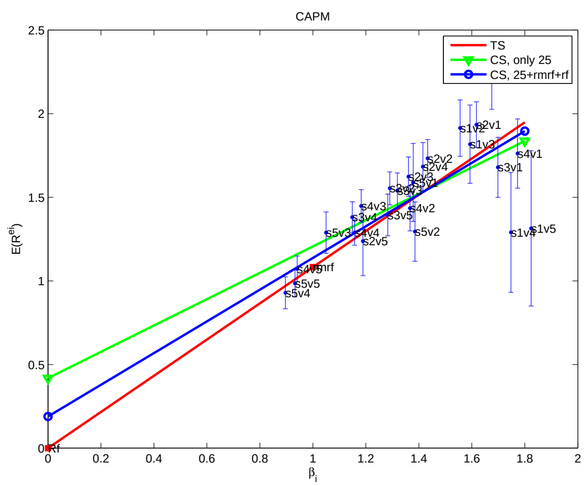
Lesson: Don't do this! Including Rf as a test asset, not allowing a free intercept, or doing GLS cross-sectional regressions all avoid this problem.

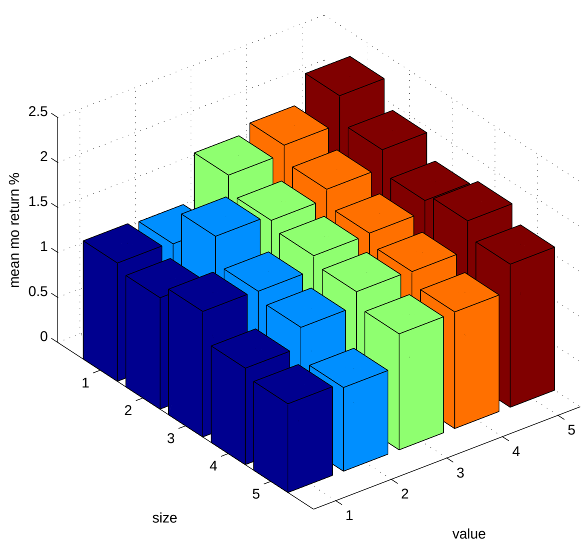
3)

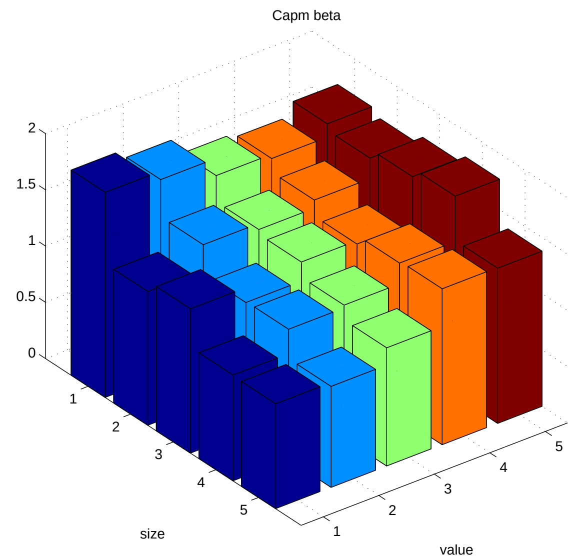
The CAPM works quite well in the earlier sample! As I look deeper into the plots (which I did not ask for) it seems that the size effect is stronger in expected returns; the small-growth anomaly is absent showing high returns there; and the value premium is perhaps a bit weaker. However, large-cap betas do rise with value, and they always rise with size. The major failing of the CAPM is that small-cap betas do not rise with value, but that's much less than the uniform decline of betas with value we saw in the later sample. I barely know what to make of the variation in expected returns across subsamples. The big news here is that *betas* change a lot across subsamples. But what do betas mean? Most of the betas we see are not "cashflow betas" the are "discount rate betas," correlations of your discount rate changes with the market discount rate.

Here's what I asked for. You can see that the $\hat{\lambda}$ estimates are about the same and the alpha statistics are about the same. This should give you some confidence that the cross sectional statistics $\sigma(\hat{\lambda})$ and the $\hat{\alpha}' cov(\hat{\alpha})^{-1}\hat{\alpha}$ idea works well when it's supposed to work well. The cross sectional method with no constant has larger $\sigma(\hat{\lambda})$ because it throws out information, there are only 23 degrees of freedom. When you put the market and risk free rate information back in, you get a $\sigma(\hat{\lambda})$ more in line with time series. It's still bigger because OLS is bigger than GLS. Interestingly the χ^2 statistic and p value are the same for the "efficient" (GLS, TS) and "inefficient" (OLS) estimate.

```
 comparison table
CAPM, Sample   193201 to   196212
              gamma  lambda s(gamma s(lambd rmse(a)   E|a|   chi2     5%     %p
TS                   1.08           0.32    0.23     0.17   47.31   37.65   0.45
CS, free g    0.42   0.79   0.42    0.52    0.22     0.17   35.75   35.17   4.38
CS, with f    0.19   0.95   0.21    0.42    0.22     0.16   47.51   37.65   0.43
```







4. Now for the FF model. My results are consistent with FF in this larger sample. The rmrf betas are all about one. Note how these *multiple regression* betas are different from the *single regression* betas above. The market, hml, and smb are somewhat correlated, so multiple regressions assign some of what seemed to be movement with the market to movement with hml. The h coefficients rise as we go to the right and the s coefficients rise as we go up. The alphas are about as in FF, except the small growth alpha is much worse. Also, large value seems to underperform . Small growth stocks *underperform* dramatically. Note that this underperformace is not so much bad mean returns – they are the same as other mean returns. It comes from the combination of mean returns and betas. To take advantage of it, you don't short small growth stocks, you have to short small growth stocks *and* invest in hml. The 3F R2 are all above 90%, leading me to label the model more APT than "mimicking portfolio for state variables".

```
Sample   196301 to   201012

3F model b
    1.0841     0.9540     0.9157     0.8820     0.9828
    1.1081     1.0111     0.9589     0.9657     1.0827
    1.0906     1.0369     0.9839     0.9800     1.0578
    1.0554     1.0759     1.0732     1.0143     1.1435
    0.9702     1.0007     0.9776     0.9898     1.0361

3F model h
   -0.3127     0.0303     0.2743     0.4441     0.6945
   -0.4026     0.1246     0.3798     0.5606     0.7947
   -0.4416     0.1767     0.4361     0.6026     0.7770
   -0.4380     0.2014     0.4486     0.5658     0.8003
   -0.3684     0.1008     0.2786     0.5948     0.7463

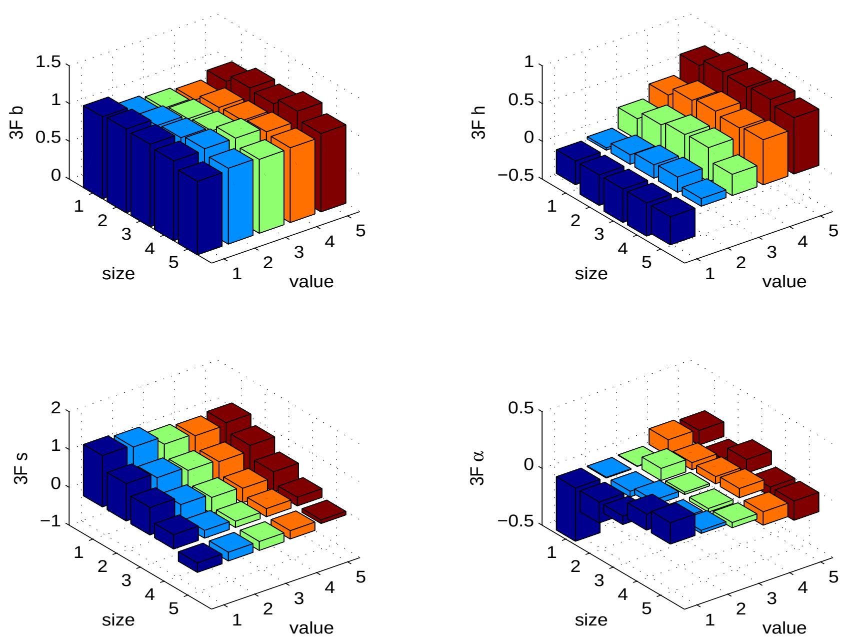
3F model s
    1.3557     1.2998     1.0892     1.0287     1.0901
    0.9852     0.8601     0.7680     0.7124     0.8596
    0.7238     0.5184     0.4242     0.3818     0.5329
```

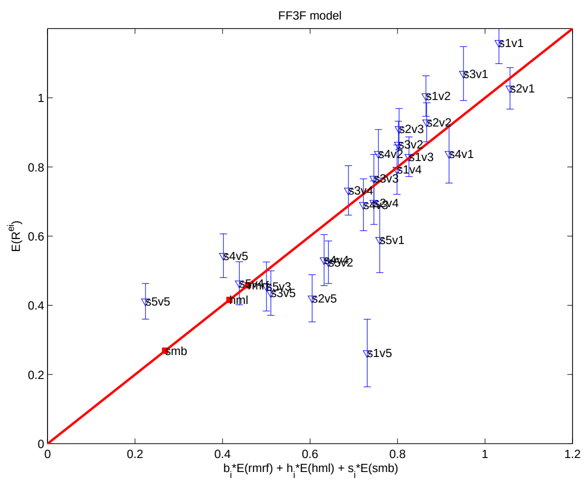
```
   0.3748    0.2069    0.1639    0.2104    0.2279
  -0.2503   -0.2301   -0.2352   -0.2178   -0.0945

3F model alphas
  -0.4688   -0.0066    0.0034    0.1400    0.1273
  -0.1846   -0.0493    0.1065    0.0623   -0.0301
  -0.0750    0.0445    0.0206    0.0628    0.1191
   0.1414   -0.1014   -0.0315    0.0817   -0.0798
   0.1876    0.0253   -0.0459   -0.1175   -0.1699

T on 3F alphas
  -4.8028   -0.0922    0.0594    2.3976    2.0969
  -2.7051   -0.7822    1.8016    1.1061   -0.5017
  -1.1643    0.6232    0.2940    0.9299    1.5274
   2.2395   -1.3712   -0.4205    1.1673   -0.9434
   3.6472    0.4068   -0.6483   -1.9126   -1.7841

3F R2
   0.9207    0.9428    0.9510    0.9432    0.9475
   0.9524    0.9407    0.9364    0.9391    0.9469
   0.9502    0.9086    0.8942    0.8969    0.8917
   0.9391    0.8909    0.8814    0.8865    0.8729
   0.9364    0.8965    0.8597    0.8944    0.8018
```





5)

FF3F:

```
chi2 statistic, N, prob value (%)       82.70    25.00 4.29e-006
10, 5, 1% p values of chi2(N)           34.38    37.65    44.31
F statistic, N, T-N-K, prob value(%)     3.15    25.00   548.00 6.70e-005
10, 5, 1% p values of F(N,T-N-K)         1.39     1.53     1.81
rms alpha,mean abs alpha,                0.14     0.10
```

As a reminder, CAPM :

```
CAPM chi2 statistic, N, prob value (%)    108.50    25.00 2.24e-010
10, 5, 1% p values of chi2(N)              34.38    37.65    44.31
F statistic, N, T-N-K, prob value (%)       4.14    25.00   550.00 2.10e-008
10, 5, 1% p values of F(N,T-N-K)            1.39     1.53     1.81
rms alpha, mean abs alpha,                  0.34     0.29
```

Interesting – another huge rejection

a) It's interesting that the test statistics for FF3F are not much better than for the CAPM. The mean absolute alpha is much lower though the rmse alpha is only about half. One huge outlier makes a difference when you square it. What's going on? $\alpha'\alpha$ can be about half as large, but $\alpha'\Sigma^{-1}\alpha$ can be about the same, if Σ falls! The R^2 is a good deal better in the FF3F time series regressions so you know Σ is a lot smaller.

All of these tests are focusing on the model's ability to price one, very poorly estimated minimum variance portfolio, formed with the Σ^{-1} matrix.

As you can see, *test statistics are not very revealing about model performance.* Now you see why Fama and French changed the rhetoric of asset pricing models away from test statistics and towards patterns in expected returns, betas, and so forth. And rightly so.

6.

```
testing for dropping factors
sample
      193201      196212
```

	rmrf	hml	smb
E(f)	1.082	0.514	0.362
t	3.352	2.247	1.933
sharpe	0.602	0.404	0.347
E(f*)	0.587	0.026	0.091
t	2.311	0.145	0.543
sharpe	0.418	0.026	0.099

In the early sample, the raw premiums are all strong, including smb. However, both smb and hml are correlated with the market. Thus, the alphas or orthogonalized premiums are zero. Thus *even though we reject the capm and we reject the 3 factor model, we "accept" the idea that hml and smb are not needed – their improvements on the capm are not significant.*

```
testing for dropping factors
sample
      196301      201012
```

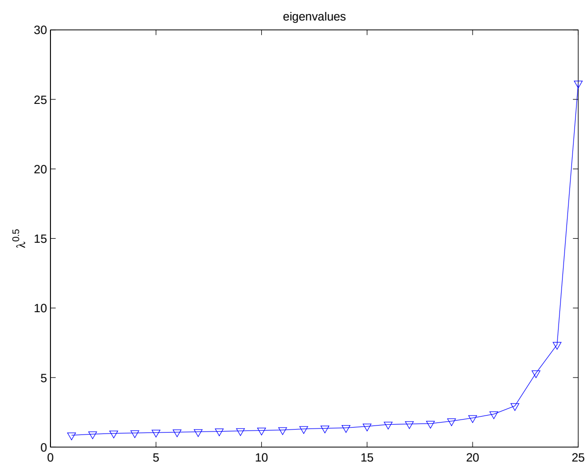
	rmrf	hml	smb
E(f)	0.458	0.416	0.269
t	2.431	3.405	2.038
sharpe	0.351	0.491	0.294
E(f*)	0.518	0.530	0.256
t	2.933	4.581	2.024
sharpe	0.430	0.665	0.298

In the later sample, not even the raw smb premium is significant. hml is a little negatively correlated with the market, so it's orthogonalized factor is even stronger than the raw factor. *For pricing purposes a two-factor model would suffice in the postwar data.*

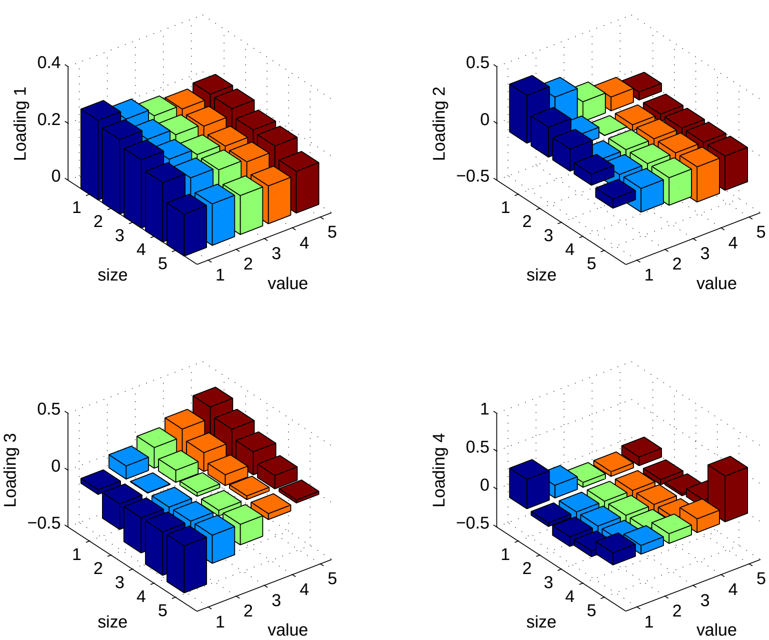
Why do FF keep three factors? My hunch is that small stocks are a very important dimension of the *covariance matrix* of returns. It remains true that small stocks all "move together". This is an important fact to keep in mind (just like the comovement of firms in different industries) even if, in the end, we decide that covariance with this sort of common movement does not give rise to any risk premium. Maybe they are an APT after all! Seriously, for short-sample risk correction and performance evaluation, it makes sense to include a huge factor even if that huge factor is not priced. It may have a good return in a short sample. Otherwise, suppose you evaluate an idea (ipos say) that has a strong "small" component. It might have a good return in a short sample. If you left smb out, you wouldn't notice that fact. *For the purpose of performance evaluation and empirical risk adjustment it may make sense to include pervasive "variance factors" even if in longer samples those factors don't really help to understand pricing.*

Finally, keeping a "useless" factor for *pricing* is still useful – it raises the R^2 in the time-series regression lowers Σ, and thus makes all the estimates more precise.

7 I plot the eigenvalues: Pretty clearly there are three "significant" eigenvalues.

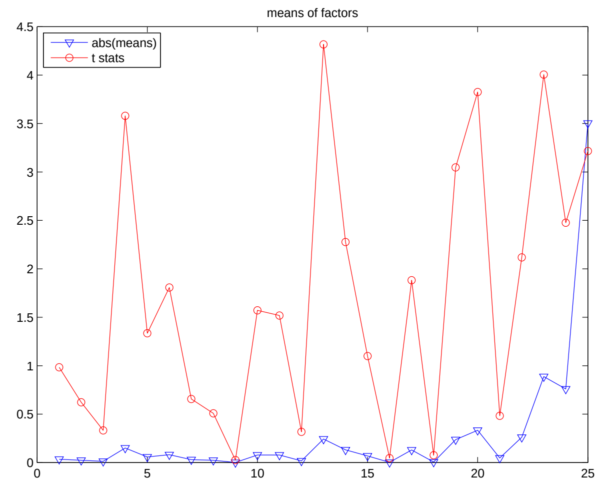


I plot the loadings. The first is the equally weighted market, with an interesting tilt towards *small* stocks. I think that's because they are most volatile, so an objective of fitting these portfolios by *variance* weights them a lot. If you weighted your objective by market cap, you'd get the market portfolio! 2 and 3 are obvious combinations of size and book/market factors. Interestingly, tha last one is small growth / long value, another instance of "where there is mean, there is covariance."

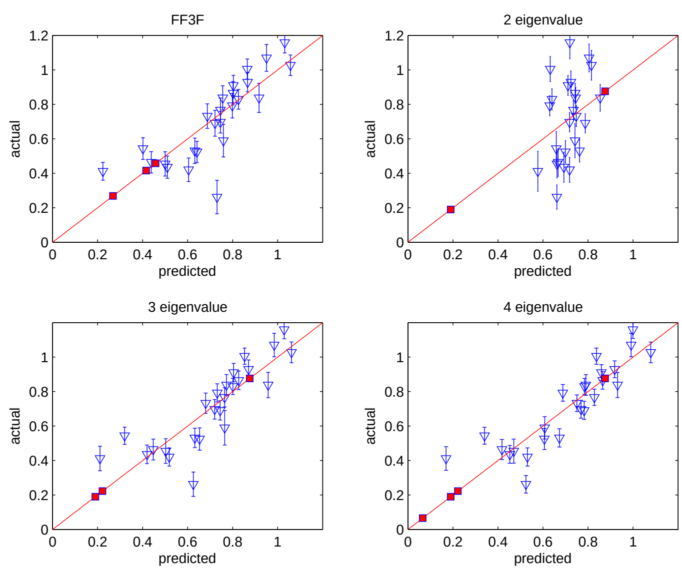


Here is the plot of means and t stats. Means by themselves are not that meaningful of course because the scale of loadings is arbitrary. Here the definition that $\sum q_i^2$ of eigenvalues helps to maintain an economically interesting scale. The t statistics and sharpes start out ok, suggesting we can ignore factors past the first 3 or 4. But then they go nuts. These are very small factors with strong + and - loadings. We have to use some economic intuition to ignore them.

We learn that factors which are small in variance are not necessarily small in sharpe ratio. Are they real, or are they like CP's tiny factors that caused rejections?



Much better statistics come from looking at actual vs predicted and alphas. Here are actual vs. predicted for FF3F and factor models using 2-4 principal components. As you can see, by the third factor, we have performance almost exactly equal or slightly better than to the FF model. No surprise, the small growth factor hleps on the small growth portfolio!



Here is the comparison of statistics. As you see, we really do need all three eigenvalues to get performance as good as FF3F. The first combination value/size factor 2 isn't enough. The 3 eigenvalue model does a very little better than FF. I also include the average R2 from the time series regressions which (obviously) gets better and better. The 4th factor model loads on the large value and small growth, interestingly enough. Then, when we add it to the mix, it eliminates the large value-small growth puzzle. (In the same way that a momentum factor eliminates the momentum puzzle.)

Again, there is no guarantee that covariances will explain alphas. That's a result, not a mathematical certainty. If it were not true of course there would be high Sharpe ratio opportunities. Thus it's wonderful to see each factor in turn beat down the alpha puzzles of the previous factor model.

Disclaimer: Of course we should be cautious in the use of too many factors. They may not be stable out of sample. Also, the size factor had questionable economics, the value factor only had FF's speculations about human capital in depressed industries, and momentum has no economics. I have no hint of the economics behind a small growth - large value factor. Thus, you should probably view it as the momentum factor, an ad hoc device that may be useful for performance evaluation, but still on shaky ground for fundamental asset pricing.

```
compare FF3F and eigenvalue models
                chi2         N      %pv       5%     rmsa      |a|       R2
 FF3F         82.697    25.000    0.000   37.652    0.136    0.099    0.913
3 FF alphas
   -0.4688   -0.0066    0.0034    0.1400    0.1273
   -0.1846   -0.0493    0.1065    0.0623   -0.0301
   -0.0750    0.0445    0.0206    0.0628    0.1191
    0.1414   -0.1014   -0.0315    0.0817   -0.0798
    0.1876    0.0253   -0.0459   -0.1175   -0.1699

 eig F2       95.582    23.000    0.000   35.172    0.217    0.186    0.893
2 factor alphas
   -0.3999    0.1621    0.1898    0.3726    0.4397
   -0.2982   -0.0214    0.1987    0.2052    0.2116
   -0.2575   -0.0159    0.0312    0.1217    0.2649
   -0.1159   -0.2318   -0.0973    0.0927   -0.0163
   -0.1658   -0.2039   -0.2078   -0.1752   -0.1525

 eig F3       77.851    22.000    0.000   33.924    0.124    0.095    0.929
3 factor alphas
   -0.3636    0.0605    0.0284    0.1516    0.1299
   -0.0982   -0.0238    0.1054    0.0580   -0.0344
    0.0156    0.0516    0.0019    0.0378    0.0851
    0.2228   -0.1016   -0.0541    0.0651   -0.1202
    0.2005    0.0160   -0.0485   -0.1290   -0.1756

 eig F4       73.060    21.000    0.000   32.671    0.113    0.088    0.941
4 factor alphas
   -0.2625    0.1039    0.0449    0.1675    0.1595
   -0.1108   -0.0724    0.0503    0.0105   -0.0517
   -0.0174   -0.0184   -0.0616   -0.0000    0.0782
    0.2029   -0.1423   -0.0924    0.0475   -0.0935
    0.2421    0.0458   -0.0157   -0.0817   -0.0180
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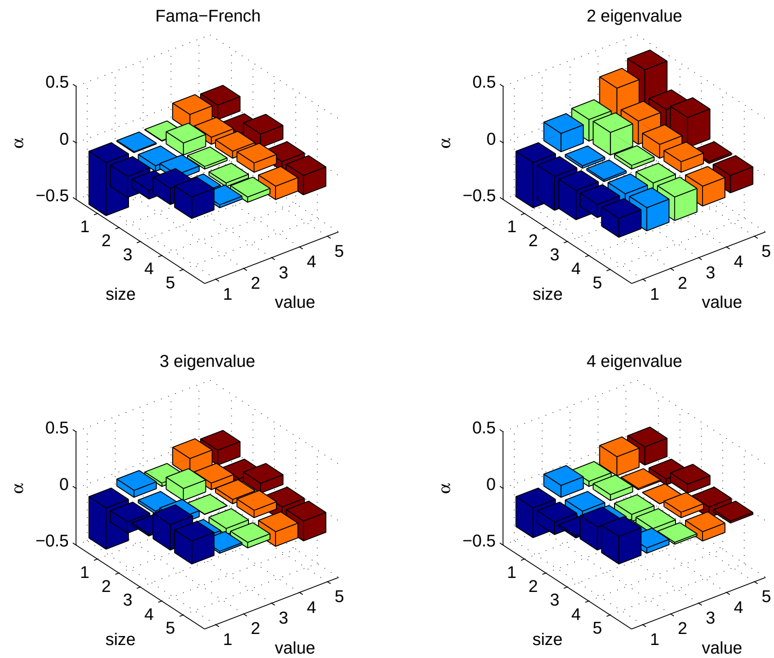
Fama−French
2 eigenvalue
3 eigenvalue
4 eigenvalue
0.5
0
−0.5
α
size
value
1
2
3
4
5

Part II

Give very short, 1-2 sentence answers. Citing page numbers and results from tables is a good idea.

Multifactor anomalies

1. Are small stocks necessarily ones with small numbers of employees, small plants, etc.?

   A: No. It's a market value sort, not a book value or other sort. Thus, it's also a 1/price kind of variable. In fact, it turns out that "small" companies, with small numbers of employees, book assets, etc., don't earn any special returns. The returns are good *only* if you define "small" in a way that involves low market prices.

2. Can we summarize Fama and French's 3 factor model amount to saying "We can explain the average returns of a company by looking at its size and book/market ratio?""

   A: NO. The *model* says you get high average returns for *covarying* with the B/M portfolio, not for *being* a high B/M firm. A firm that *was* value but *acted* like growth should get the growth premium.

3. Which gets better returns going forward, stocks that had great past growth in sales, or stocks that had poor past growth in sales?

   A: Poor – see Table III.

4. What pattern of betas explains the average returns of stocks sorted on sales growth?

   A: Table III it's mostly a value effect.

5. Are the s coefficients on sales growth portfolios constant? Can you think of a story to explain them?

   A: they are U shaped. The easiest way to get in a tail portfolio is to have a lot of variance. Small stocks have more variance

6. Which results show the "long-term reversal" effect in average returns best? Which show the "momentum" effect best?

   A: Table VI, 60-13 since they leave out the momentum part. 12-2 shows momentum best, note it doesn't work so well pre 63.

7. Why do the sorts in Table VI stop at month -2 rather than go all the way to the minute the portfolio is formed?

   A: Any measurement error is then common to sort and returns, inducing the false appearance of reversion.

8. Why might the average investor try to avoid holding value stocks, and hence drive up the equilibrium premium (according to Fama and French)?

   A: p. 77. They emphasize human capital rather than wages, because people with generic skills don't lose if their companies lose, they just get jobs elsewhere. They speculate that people with jobs in value companies have a harder time relocating if their companies go down (machinists) while those in growth companies can more easily move (programmers.)

Dissecting Anomalies

1. FF point out dangers of the common practice of sorting stocks by some variable, and then looking at the average returns of the 1-10 spread portfolio. What don't they like about this practice?

   A: 1654. Their main complaint is that these portfolios are equal weighted, thus focusing on tiny stocks.

2. How do FF define "tiny" stocks? What fraction of their sample are tiny, and what fraction of market cap do tiny stocks represent? How can the

   A: 1656 breakpoints are 20 and 50 percentiles of NYSE, 60% of stocks and 3% of market value. The sample includes amex and nasdaq which have many smaller stocks than NYSE.

3. Are the average returns in Table II adjusted for the three-factor model somehow?

   A: They are "characteristic-adjusted", explained 1658 below II. sorts. This means, find the portfolio of 25 size/book/market whose size and B/M are closest, and subtract off that return. The text says that true size and book/market alphas gives similar results, though since there are some big alphas (small/growth) separating average returns and betas in the 25, I'm not altogether convinced. OTOH, FF argue that individual-stock hml, smb betas are measured badly and wander over time. Thus, they say, the characteristic is a better measure of beta than beta itself. Anyway, read the table as FF's ideas about alphas after controlling for size and b/m.

4. Why are the t- statistics for the High-Low portfolio in Table II so much better than for the individual portfolios?

   A: We're really not that interested in whether portfolio excess returns are different from zero. We want to know if they're different *from each other*. If all averages were equal to each other but different from zero, it wouldn't be that interesting. Each portfolio could be within a standard error of zero, but if the long-short portfolio is significant, you have a trading strategy/anomaly.

5. It seems we get better returns and higher t statistics the finer we chop portfolios. Can you make anything look good by making 100 portfolios and then looking at the 1-100 spread?

   A: No. First, you're sorting on microcaps which you may not trust. More importantly, the variance goes up as well, so the sharpe ratio E/σ and the t statistic $E/\left(\sigma\sqrt{T}\right)$ should stabilize as you get more extreme. (This is shown in lecture)

6. Name an anomlay that only seems to work in tiny stocks in Table II.

   A: Asset growth.

7. The Profitability sort seems not to work in Table 2.. How did people think it was there? (Hint: 1663 pp2)

   A: 1663 pp2 *With controls for cap and B/M.* There is a profitability effect on its own, but size and B/M pick it up. This is a good instance of the point of the paper – what works *in the presence of the others*, what has *marginal* power, what is the *multiple* regression forecast of returns, not each variable at a time.

8. Explain why the numbers in Table III jump so much between 4 and high.

   A: The 1/5 of extreme values of any distribution is way spread out. Table III momentum lets you make the connection between autocorrelation and momentum – look at the past returns!

9. Explain what the top left 4 numbers mean in Table IV.

   A: These are regression coefficients. You're seeing the basic size and B/M effects in expected returns. Larger size means smaller ER, Larger B/M means larger ER.

10. "The novel evidence is that these results [size effect] draw much of their power from tiny stocks" What numbers in Table IV are behind this conclusion?

    A: This is the disappearance of the size coefficient in the other groups in the top left part of Table 4. Note size is also much weaker post 1979 – when the effect was published and small stock funds started. (not in this paper)