FX

- We're going to review the current state of FX work. In my opinion it's a good bit behind the stock and bond work above, which means low-hanging fruit for you to apply the same ideas. (Note Lustig et. al. as a "low hanging fruit" example.")

- The basic idea: Suppose the UK interest rate = 5%, US interest rate = 2%. Should you invest in UK? The naive view: Yes, you'll make 3% more. The traditional view: No, the pound will depreciate 3% (on average) The fact: The pound seems to go *up!* As with D/P the adjustment goes the "wrong way."

- Evidence: Typically country by country time series regressions, following Fama's original in 1981.

$$\$ \text{ Return}_{t+1}^{i} = a_i + b_i(i_t^f - i_t^d) + \varepsilon_{t+1}$$

$b \geq 1$. Not even zero. Small R^2.

- This is the basis for the "carry trade," borrow in low interest rate countries and lend in high interest rate countries. (Note the analogy to "ride the yield curve" also called a "carry trade," "borrow in low interest rate *maturities* and lend in high *maturities*. I long for a unifying view!)

- Variations: You can do this for two common choices of *right hand* variables, forward-spot spread or interest differential, and you can do this for two choices of *left hand* variable, exchange rate or excess return. As usual, everything is related by identities. The identies (or arbitrage) say that two ways of getting money to the same place give the same result.
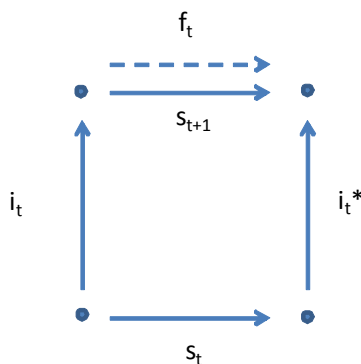

Figure 9:

1. "Covered interest parity" and a right hand variable identity.

$$f_t - s_t = i_t^* - i_t$$

i.e. going around the box gets zero

$$i_t + f_t - i_t^* - s_t = 0$$

Thus, regressions with $f_t - s_t$ on the right are the same as regressions with $i_t^* - i_t$. (My units. Dollar per foreign currency. If f and s_{t+1} are bigger numbers, that means you get more foriegn currency per dollar in the future. That's the same as earning a big interest rate abroad. 1Euro/Dollar today and 2Euro/dollar next year – dollar appreciates, $f_t, s_{t+1} > s_t$,: Hold a euro for a year at zero i^* is the same as losing 50% on dollars. Euro rates have to rise to compensate. If you want Dollars/Euro add negatives appropriately)

2. Left hand hand variable identity: You can look at exchange rate changes (Δs) or expected returns. By an identity,

$$rx_{t+1} = i_t^* - i_t - (s_{t+1} - s_t)$$

Thus, if you regress on either right hand variable,

$$b_r + b_s = 0$$

Either exchange rates are predictable or excess returns are predictable.

- Like bonds the first question was "does expectations work?" Is $f_t = E_t(s_{t+1})$? Fama figured out to do this by a) running s_{t+1} on f_t b) much better, running $s_{t+1} - s_t$ on $f_t - s_t$ as a much more powerful test c) once you see that isn't working, it's interesting to note rx_{t+1} on $f_t - s_t$ d) I like expressing it in terms of $i_t^* - i_t$ which is more intuitive to me than forward rates.

- Preview: The CP common factor has not been done, "covariance with what" is only beginning, the present value relation hasn't been done, etc.

- Fact 1: Expectations is about right in *levels* (433). (Just as the yield curve is pretty flat *on average*)

- Fact 2: We whould see $b_s = +1$ in regressions. In fact: it's negative. Some numbers. From *Asset Pricing*

Table 20.11.

	DM	£	¥	SF
Mean appreciation	−1.8	3.6	−5.0	−3.0
Mean interest differential	−3.9	2.1	−3.7	−5.9
b, 1975–1989	−3.1	−2.0	−2.1	−2.6
R^2	.026	.033	.034	.033
b, 1976–1996	−0.7	−1.8	−2.4	−1.3

The first row gives the average appreciation of the dollar against the indicated currency, in percent per year. The second row gives the average interest differential—foreign interest rate less domestic interest rate, measured as the forward premium—the 30-day forward rate less the spot exchange rate. The third through fifth rows give the coefficients and R^2 in a regression of exchange rate changes on the interest differential = forward premium,

$$s_{t+1} - s_t = a + b(f_t - s_t) + \varepsilon_{t+1} = a + b(r_t^f - r_t^d) + \varepsilon_{t+1},$$

where s = log spot exchange rate, f = forward rate, r^f = foreign interest rate, r^d = domestic interest rate.
Source: Hodrick (1999) and Engel (1996).

An update from Burnside et al.

	1 Month Regression			3 Month Regression		
	α	β	R^2	α	β	R^2
Belgium†	-0.002	-1.531	0.028	-0.005	-0.625	0.008
	(0.002)	(0.714)		(0.006)	(0.669)	
Canada	-0.003	-3.487	0.045	-0.007	-2.936	0.072
	(0.002)	(0.803)		(0.005)	(0.858)	
France†	0.000	-0.468	0.004	0.001	-0.061	0.000
	(0.002)	(0.589)		(0.005)	(0.504)	
Germany†	-0.005	-0.732	0.005	-0.012	-0.593	0.007
	(0.003)	(0.704)		(0.008)	(0.650)	
Italy†	0.005	-0.660	0.010	0.008	-0.012	0.000
	(0.002)	(0.415)		(0.006)	(0.392)	
Japan*	-0.019	-3.822	0.030	-0.063	-4.482	0.100
	(0.005)	(0.924)		(0.014)	(1.017)	
Netherlands†	-0.009	-2.187	0.029	-0.018	-1.381	0.026
	(0.004)	(1.040)		(0.009)	(0.816)	
Switzerland	-0.008	-1.211	0.012	-0.020	-1.050	0.022
	(0.003)	(0.533)		(0.008)	(0.536)	
USA	-0.003	-1.681	0.017	-0.008	-1.618	0.037
	(0.002)	(0.880)		(0.006)	(0.865)	

TABLE 2

UIP Regressions, 1976-2005

* Data for Japan begin 7/78
† Data for Euro legacy currencies ends 12/98
Notes: Regression of [S(t+1)/S(t)-1] on [F(t)/S(t)-1]. Standard errors in parentheses.

Table I
UIP Regressions.

This table reports coefficient estimates from the regression of the time $t+1$ log currency return on the time t log forward premium,

$$s_{i,t+1} - s_{i,t} = a_0 + a_1 \cdot (f_{i,t} - s_{i,t}) + \varepsilon_{i,t+1} \qquad H_0 : a_0 = 0, \, a_1 = 1$$

Currency returns are computed using 21-day rolling windows and span the period from January 1990 to December 2007 (all exchange rates are expressed in terms of dollars per unit of foreign currency). The forward premia are measured using the spread between one-month eurocurrency (LIBOR) rates for loans denominated in U.S. dollars and loans denominated in the foreign currency. The table reports regression coefficients, standard errors (in parentheses), and the χ^2 test statistic for the null hypothesis of UIP (p-values in parentheses). Standard errors in individual regressions are adjusted for serial correlation using a Newey-West covariance matrix with 21 lags (1990-2007: N = 4,673; 1999-2007: N = 2,542). The pooled (panel) regression is run with country-fixed effects; the reported standard errors are robust to within time-period correlation and are adjusted for serial correlation. The pooled regression χ^2 statistic is computed for the null that all country fixed effects are zero and the intercept is equal to one. R^2_{NFE} reports the adjusted R^2 from the panel regression net of the fixed effects (1990-2007: N = 42,057; 1999-2007: N = 22,878). XS reports the time series means and standard errors of the regression coefficients from cross-sectional regressions performed for each t. For the cross-sectional regressions R^2 is the mean adjusted R^2 (1990-2007: N = 4,673; 1999-2007: N = 2,324).

| Currency | 1990-2007 | | | | 1999-2007 | | | |
	$\hat{a}_0$	$\hat{a}_1$	R^2_{NFE}	χ^2 test	$\hat{a}_0$	$\hat{a}_1$	R^2_{NFE}	χ^2 test
AUD	-0.0021	-1.6651	0.0087	9.05	-0.0021	-3.6789	0.0240	9.40
	(0.0023)	(1.0487)		(0.01)	(0.0036)	(1.9450)		(0.01)
CAD	0.0006	-0.1754	0.0000	6.87	0.0038	-1.1337	0.0016	5.93
	(0.0010)	(0.5356)		(0.03)	(0.0016)	(2.1465)		(0.05)
CHF	0.0027	-1.2094	0.0060	5.20	0.0100	-4.3559	0.0312	8.64
	(0.0024)	(0.9961)		(0.07)	(0.0041)	(1.8256)		(0.01)
GBP	0.0006	0.0591	-0.0002	1.28	0.0042	-4.3155	0.0401	11.04
	(0.0016)	(0.8981)		(0.53)	(0.0023)	(1.6417)		(0.00)
EUR	0.0021	0.7291	0.0021	3.88	0.0002	-1.6417	0.0053	4.60
	(0.0018)	(1.2675)		(0.14)	(0.0023)	(1.7499)		(0.10)
JPY	0.0059	-1.9686	0.0146	11.83	0.0048	-1.5828	0.0070	5.18
	(0.0025)	(0.8638)		(0.00)	(0.0045)	(1.3785)		(0.08)
NOK	0.0018	0.7030	0.0052	1.88	0.0017	-1.0765	0.0049	5.42
	(0.0017)	(0.6196)		(0.39)	(0.0025)	(1.2000)		(0.07)
NZD	-0.0045	-2.4182	0.0131	15.58	-0.0067	-4.7480	0.0406	16.88
	(0.0034)	(1.1961)		(0.00)	(0.0045)	(1.6800)		(0.00)
SEK	0.0006	0.6554	0.0053	0.53	0.0031	-3.2701	0.0327	10.35
	(0.0017)	(0.5966)		(0.77)	(0.0023)	(1.3594)		(0.01)
Pooled	FE	-0.0986	0.0000	7.1022	FE	-2.7863	0.0190	35.41
		(0.6500)		(0.72)		(1.1179)		(0.00)
XS	0.0007	-0.1622	0.1161	-	0.0017	-0.5154	0.1157	-
	(0.0003)	(0.0853)			(0.0005)	(0.1087)		-

- "On average" vs. "for countries."

$$R^i_{t+1} = a_i + b(i^*_t - i_t) + \varepsilon_{t+1}$$

We should see this as a panel data regression. The issue, just like credit spreads above. a) country dummies? b) time dummies c) common factors on the right hand side?

- Can we see it in a graph just like we did for bonds?
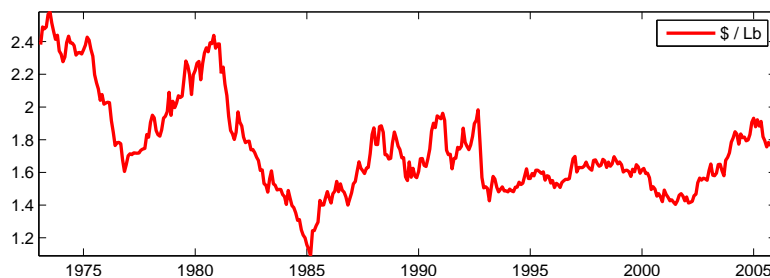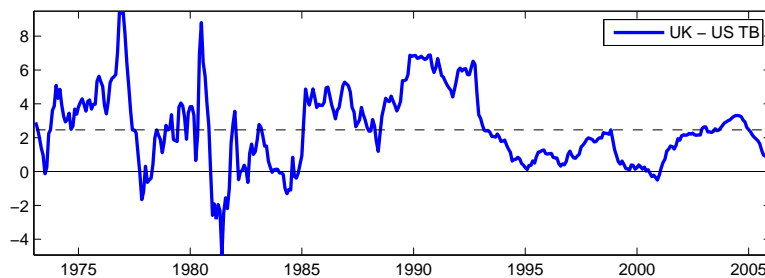
1. Higher interest rates *are* associated with stronger exchange. \$/LB goes up when UK rate goes up. There is something to the standard story!

2. If you invest in higher UK rates, you make money until pound weakens, until \$/UK goes down.

3. A higher exchange rate goes on *for many years at a time.* These are 3 month rates. You see the same "sluggish adjustment" as in yields.

- Comments:

   1. "Carry trade" by $i^* - i > 0$ and $i^* - i > E\left(i^* - i\right)$ are very different! (See hoizontal lines in the graph). The right hand variable is very slow moving.

   2. The R^2 is low (monthly data). It's economically large: *All* interest differential (and more?) is expected return, *none* expected depreciation (≤ 1 year). Again, read the regression as "what is the information in the price" not "how do I start my hedge fund?"

   3. NB though, many hedge funds do essentially this. As in CP1 they usually trade "enhanced carry," they have some idea of "when to get out." They also form portfolios (something like $\Sigma^{-1}(i - i^*)$)

   4. Economics? Low interest rate episodes are recessions, so this has the usual business cycle pattern. When the US risk premium is high, so is the premium for holding currency risk.

   5. Wait, that was too quick.

      (a) Why does $cov(u'(c_{t+1}^{us}), \text{euro}_{t+1}/\$_{t+1}?)$ (And the opposite for the Euro investor.) Models have some work to do.

(b) If so, we should not have *separate* regressors $i^* - i$ for each country, there should be a common factor (maybe $\frac{1}{N}\sum_j i_j^* - i_t$) that forecasts *all* currencies. Do like CP1 for currencies? It hasn't been done yet really.

(c) If so, $i^* - i$ should forecast stock and bond returns, as $\gamma' f$ forecasts stock returns. And DP should forecast bond and FX returns. and $\gamma' f$ should forecast FX...Or there should be a reduced factor structure in which a common component of $i^* - i$, $DP, \gamma; f$, forecasts a common component of all returns.

6. a_i is important.

$$\$ \text{ Return}_{t+1}^i = a_i + b_i(i_t^i - i_t^{us}) + \varepsilon_{t+1}^i; \ t = 1, 2, ...T$$

There are "country dummies" in the regression. If you leave out a_i you get Turkey or Brazil – perpetually high i^*-i (40%) matched by 40% inflation and devaluation. The fact is "more than usual" interest differential corresponds to a high return. Does this matter? Does slow moving right hand variable mean the a_i estimate biases b_i up? If there is a unit root in inflation, maybe the a_i is meaningless, there is no "usual" differential. Paper topic! (We'll find that "portfolio dummies" vs. "time dummies" makes a big difference in looking at stocks too, see "Discount rates" discussion of Fama and French.

7. The graph suggests "expectations works" in longer run regressions. I'm not aware of papers that do the right hand panel of the Fama-Bliss table well to document this well. (more low hanging fruit.)

8. What about the "present value identity?" Does this link to longer–term regressions? Here is a stab at the question.

$$\begin{aligned}
rx_{t+1} &= i_t^* - i_t - s_{t+1} + s_t \\
s_t &= (i_t - i_t^*) + rx_{t+1} + s_{t+1} \\
s_t &= E_t \sum_{j=0}^{\infty} \left[(i_{t+j} - i_{t+j}^*) + rx_{t+j+1} \right] + E_t s_{t+j}
\end{aligned}$$

This isn't much help, I'm not getting any discounting and nominal exchange rates can go anywhere. But how about real exchange rates? Let $\pi_t =$ inflation, so real exchange rate change is

$$s_{t+1}^r - s_t^r = s_{t+1} - s_t - \left(\pi_{t+1}^* - \pi_{t+1} \right).$$

Then,

$$\begin{aligned}
rx_{t+1} &= i_t^* - i_t - \left(s_{t+1}^r - s_t^r + \left(\pi_{t+1}^* - \pi_{t+1} \right) \right) \\
rx_{t+1} &= (i_t^* - i_t) - \left(\pi_{t+1}^* - \pi_{t+1} \right) - \left(s_{t+1}^r - s_t^r \right) \\
rx_{t+1} &= (r_t^* - r_t) - \left(s_{t+1}^r - s_t^r \right)
\end{aligned}$$

where $r =$ ex-post real rates.

$$\begin{aligned}
s_t^r &= (r_t - r_t^*) + rx_{t+1} + s_{t+1}^r \\
s_t^r &= E_t \sum_{j=0}^{\infty} \left[(r_{t+j} - r_{t+j}^*) + rx_{t+j+1} \right] + E_t s_{t+j}^r
\end{aligned}$$

So, if the expected real exchange rate must approach one in the long run, the current real exchange rate must be matched by real interest differentials or excess returns. I wonder which it is....

9. Big crashes – the "peso problem" was invented precisely for this regression! Many governments do "soft interventions" leading to big left tails and long samples that don't include the left tails.

10. *Strategies that involve small constant gain and occasional big crashes are ubiquitous in hedge funds etc. Dynamic trading can synthesize options. Earth quake insurance. Put options. This is very hard to tell by statistical measures.*

11. The last two comments motivate Jurek, Burnside Eichenbaum Rebelo "crash neutral" currency trades.

Jurek

(April 2009 draft) Good: Updates, it investigates the "peso problem," and the claim that UIP was profitable even if you bought crash insurance against peso problems. It extend pricing questions to currency options. I like to read a recent paper for the latest data, and som reassurance on the "state of the art," so if you do better than this you're doing better than a big literature.

Bad: In many ways it's an example of "how not to write a paper." It's a train of thought and travelogue of experiments. Table IX *is* the paper. If the data kill you in revision, you have to rewrite the paper, not treat it as an "update." Writing papers is the art of throwing things out.

1. Table 1: The standard regressions. Note the much better performance in the later priod. Note the small R2.

2. Table II "carry trade" portfolio returns. At least it's good to look at some portfolios across currencies rather than currency by currency regressions. The basic portfolio one is just long/short depending on the sign of $i - i^*$, ignoring the amount. SPR is proportional to the amount of $i - i^*$. (Why not a real portfolio, $\Sigma^{-1}\mu = \Sigma^{-1}\left(a + b\left(i - i^*\right)\right)$?) Note the high Sharpe ratios. Note that *Portfolio returns can look good with low R^2, like momentum! Portfolios are a way to look at $E(R^e)/\sigma(R^e)$ not $var(bx_t)/var(R^e)$,* and the former is more interesting.

3. "Carry" = average interest differential. This is large – warning! This suggests that there is one data point! *How much of these "profits" are just sitting in one currency through the whole sample = 1 data point? See Figure 3*

4. Figure 1: returns are scaled to same volatility. Note the big crash! Is it over?

5. Crash-neutral construction. P. 12 bottom use out of the money puts, but scale up the portfolio so at the money it has the same sensitivity to exchange rate variations. See Figure 5. Thus, "simultaneously decreasing exposure to depreciations of the high interest rate currency and increasing exposure to its appreications" (p 12)

6. Table VII crash-neutral trades for portfolios through 2007. Note the decline in mean. relative to T II. But there is not so much decline in Sharpe ratios. Jurek: "declines represent 30-40% of the return to the unhedged strategy." This is where people got the idea that the carry trade worked even if you buy peso-problem protection. (Note a similar story. Before October 1987, out of the money equity put options were cheap. After that date, they rose a lot and have stayed high ever since!)

7. Figure 8 the crash-neutral trades in the crash. Why did even crash protected decline? I thought they were crash protected? The price of put option protection shot up in the last few months.

8. Table 8 The quarterly protection seems to be doing better, echoing my story about Figure 8. But are you allowed to search ex-post over the protection horizon?

9. Conclusion: a muddle. Is anything left of crash-protected returns?

Lustig, Roussanov, and Verdelhan

- Read Up through p. 15 only (April 2009).

- Big picture. Rather than run regressions, sort in to portfolios and look at means; then eigenvalue decompose the portfolio covariance matrix. "Do like Fama-French" (and a bit "like Cochrane Piazzesi") for FX rather than regressions.

- It makes the connection between regression and Fama-French procedures. It starts some extended musing for me on "how should we characterize $E_t R_{t+1}$ and $cov_t (R_{t+1}, f_{t+1})$?

- (Background. Look quickly at Fama French 1996 Table 1)

- Table 1. Average returns in portfolios sorted on the basis of $f_t - s_t = i^* - i$ across countries.

  1. Where are the standard errors?
  2. Please put t subscripts ($\Delta s_{t+1}, rx_{t+1}$ but $f - s_t$)!
  3. The Point: high $f - s$ correspond to positive appreciation, not negative, and hence to positive excess returns.
  4. It's nice to put bid-ask spreads in.
  5. High-low portfolios: we really want to know whether $E(R)$ is *different* across portfolios, and this is a simple way to do it. (More thoughts coming on this below.)

- Table 2: Principal components of protfolios.

  1. No surprise "$ moves" is the first component.
  2. The second component is "slope" and third is "curve".

- Table 3: FF style portfolios HML and EW market. This is a cross sectional regression of $ER = \beta\lambda$ with prespecified factors.

    1. Betas panel II: Market betas are all about 1 and hml betas rise. We knew this from the covariance matrix

    2. It's not a tautology. *The high interest rate countries will all fall or rise together.* It didn't have to happen

    3. Factor risk premiums. Of course, we account for cross sectional variation with a big λ on HML. You see the pattern that ER is higher in the high $f - s$ portfolios, and the betas are high there as well.

- Table 5. This is better. It uses the first two principal components as factors. d is the first component, c is the second.

- *UIP risk premium is earned for covariance with the "slope" factor.*

- This had to be. We have a pattern of - to + in the expected returns, so a successful factor had to be - to + as well.

- Is this different from Cochrane Piazzesi who find covariance with the "level" factor?

    1. No. CP had b>0 – the b were all positive. So of course it had to be a level factor.

    2. No, because CP are examining *bonds,* and here they're examining *portfolios.* CP model

    $$E_t rx_{t+1}^{(n)} = b_n x_t$$

    Thus, when x_t is positive, CP will put long bonds in portfolio 1 and short bonds in portoflio 5. However, when x_t is negative, CP will put *short* bonds in portoflio 1 and *long* bonds in portfolio 5. Covariance of *bond* returns with a level shock to *yields* is not the same thing as covariance of *portfolio* returns (which change composition) with a level shock to those *portfolio returns.* I suspect if we do the LRV procedure on CP bond data we would get exactly LRV's results. And vice versa? A good problem set/ paper question!

- **Preview on portfolios and time series regressions:**

    1. Now, rthink about what they're doing – *relation between portfolio sorts and regressions.*

    2. Asset pricing is in the end about $E_t(R_{t+1}^{ei}) = cov_t\left(R_{t+1}^e, f_{t+1}\right)\lambda_t$. $R_{t+1}^e = a + bx_t + \varepsilon_{t+1}$ tells you about $E(R_{t+1}^e|x_t)$, and forming portfolios based on x_t also tells you $E(R_{t+1}|x_t)$. It's really a non-parametric forecasting regression with a rather inefficient kernel!

    3. Similarly, the FF procedure amounts to $cov_t\left(R_{t+1}^e, f_{t+1}\right)\lambda_t = cov\left(R_{t+1}^e, f_{t+1}|x_t\right)\lambda$. *FF portfolios are a brilliant way to reduce a time-varying conditional problem to an unconditional problem.*

    4. Of course, we've done this many times before, i.e. $0 = E_t(m_{t+1}R_{t+1}^e); 0 = E(m_{t+1}R_{t+1}^e \otimes z_t) = E(m_{t+1}\left[R_{t+1}^e \otimes z_t\right])$

5. Basically, but *the details really matter.* Is time series or cross sectional variation more important? Is the *value* of x_t or the *portfolio rank* more important?

• The cp "common factor" investigation?

Closing FX/predictability thoughts

• A common pattern across all assets:

   1. Dividend yield forecasts stock returns
   2. Long yield - short yield forecasts long-short bond returns
   3. Foreign - domestic yield forecasts foreign - domestic returns
   4. (Cross section – B/M forecasts returns. In this case, both returns and earnings)

• More facts in common with stocks, bonds

   1. "Follow yield," "All price variation = Expected returns"
   2. "Missing adjustment" (short run, i.e. ≤ 1 year)
   3. Expected returns are high in "Bad times", P/D is low, R^f is low relative to R^{f*}, and R^f is low relative to y^{long}.
   4. Is there a "single recession related factor?" R^f, term spread, bond forecast factor also forecast stock excess returns. Do they forecast fx? does $i - i^*$ forecast stock returns? Etc.

• More puzzles in international finance

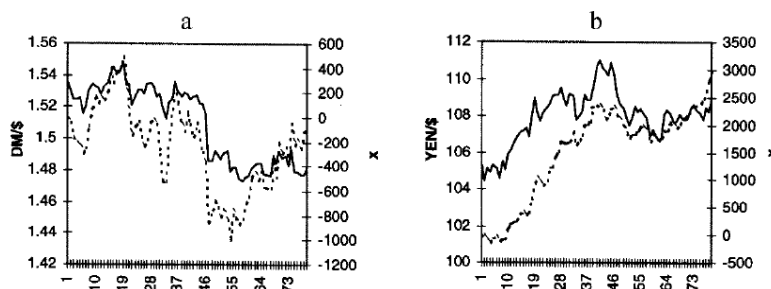   1. News—flow and price correlations

FIG. 1.—Four months of exchange rates (solid) and cumulative order flow (dashed), May 1–August 31, 1996: *a*, deutsche mark/dollar; *b*, yen/dollar.

   2. Fact: *Net* order flow *is associated with* price changes. ("order flow" not "trades")
   3. Don't jump to: *Any* order *causes* price changes. (Brandt and Kavaiecz coming up)

Brandt, Cochrane, Santaclara

Puzzles

in international finance pretty much define the field.

1. UIP (Just studied): $i^* - i$ seems to imply appreciation, not depreciation, at least for a while, and corresponding profits.

2. The volatility of exchange rates; correlation of real/nominal exchange rates.

$$\sigma \left(\ln \frac{e_{t+1}}{e_t} \right) = 15\%$$

   This is not matched 1-1 by inflation, so *real exchange rates vary a lot.*

   (a) (Mussa) Real relative prices across borders change when exchange rates change, suggesting "sticky" nominal prices.

   (b) When countries move from floating to fixed, relative prices across countries (sausage in Munich/Pizza in rome) become more stable, and relative prices of tradeables/nontradeables (Pizza in Rome/Oil in Rome) become more stable. Stock prices are a relative price too – installed vs. uninstalled capital. This is more dramatic.

   (c) Put bluntly, why did this happen? (See pound plot)

Figure 10:

3. Savings = investment and poor risk sharing across countries. One is about allocation across time, the other about allocation across states.

   (a) Permanent income logic means that temporary high income should be exported and then returned later. Also good news about future output (China opening) should lead to a consumption boom and huge imports of capital. Instead, China finances investment from domestic savings and exported the whole time. (This is an open economy facing world interest rates. If the whole world sees a boom, interest rates rise.) ("Feldstein-Horioka puzzle")

(b) Complete markets, Pareto-Optimum means

$$\max E\left[\lambda_1 \sum \beta^t u(c_{1t}) + \lambda_2 \sum \beta^t u(c_{2t})\right] \text{ s.t. } c_{1t} + c_{2t} = C_t$$

$$
\begin{aligned}
\lambda_1 \beta^t u'(c_{1t}) &= \lambda_2 \beta^t u'(c_{2t}) \\
\beta \frac{u'(c_{1t})}{u'(c_{1t-1})} &= \beta \frac{u'(c_{2t})}{u'(c_{2t-1})} \\
\left(\frac{c_{1t}}{c_{1t-1}}\right)^{-\gamma} &= \left(\frac{c_{2t}}{c_{2t-1}}\right)^{-\gamma}
\end{aligned}
$$

In fact, $cor(\Delta c_i, \Delta c_j)$ is small (numbers follow). Worse, consumption correlations are less than output correlations.

(c) In both cases, I find it hilarous that as the world starts to look more like our models, people think this is a problem. "Global imbalances" is the buzzword for the idea that we need to *slow down* trade surpluses and deficits. Mortgage backed securities did a great job of sharing risk around the world.

4. "Home bias" in portfolios. US people hold mostly US equities, UK people hold more UK equities and so forth. This is only a puzzle however relative to a world capm model, in which the investor has no job, cares equally about consumption from all countries, etc. There are lots of easy reasons it's optimal for portfolios to focus on your own country, as in my Earth vs. Mars example.

5. Currency crashes, panics, etc. (Much silliniess; "contagion" "capital flight." etc.)

6. International is RIPE for work, as witnessed by Lustig et al and this paper. Simple models are making big progress.

This paper preview:

- We can connect domestic and foreign discount factors by a simple change of units.

$$
\begin{aligned}
M^f_{t+1} &= M^d_{t+1} \frac{S_{t+1}}{S_t} \\
\frac{utils_{t+1}}{Euro_{t+1}} &= \frac{utils_{t+1}}{\$_{t+1}} \frac{\$_{t+1}/E_{t+1}}{\$_t/E_t} \\
m^f_{t+1} &= m^d_{t+1} + \ln \frac{S_{t+1}}{S_t}
\end{aligned}
$$

Equivalently,

$$
\begin{aligned}
1 &= E(M^d_{t+1} R_{t+1}) \\
&= E(M^d_{t+1} \frac{S_{t+1}}{S_t} \frac{S_t}{S_{t+1}} R_{t+1}) \\
&= E(M^f_{t+1} R^f_{t+1})
\end{aligned}
$$

where R^f = any return (domestic or foreign) expressed in foreign currency. This is cool! Exchange rates let you see mrs, directly, ex post! Well, they let you see *differences* in mrs, but that's something.

- Important – distinguish the "discount factor for returns expressed in domestic currency" from "discount factor that only prices domestic returns." The latter makes no sense unless markets are segmented somehow. *All* we're doing here is saying that we can find a discount factor that predicts a *given* set of returns converted to Euros from a discount factor that prices *the same* returns expressed in dollars. (Which is, when you see it, rather trivial.) We are *not* constructing a discount factor that prices Euro stocks from a discount factor that (only) prices dollar stocks.

- You can do the same thing with real vs. nominal discount factors. Just mulitply and divide by π.

- Now,

$$\ln \frac{S_{t+1}}{S_t} = m_{t+1}^f - m_{t+1}^d$$

$$\sigma^2 (s_{t+1} - s_t) = \sigma^2 \left(m_{t+1}^f \right) + \sigma^2 \left(m_{t+1}^d \right) - 2\rho\sigma(m)\sigma(m)$$

- What does it take to fit the facts?

$$\sigma \left(\Delta s_{t+1} \right) = 15\%$$

1. Asset pricing, "risk sharing is better than you think." From asset markets, $\frac{E(R^e)}{\sigma(R^e)} <\approx \sigma(m)$ we need at least $\sigma(m) = 50\%$. Since $\sigma(m)$ is much bigger than $\sigma(e)$ we need a lot of positive corrleation.

$$
\begin{aligned}
0.15^2 &= 2 \times 0.50^2 - 2 \times \rho \times 0.50^2 \\
0.0225 &= 0.50 (1 - \rho) \\
0.045 &= (1 - \rho) \\
\rho &= 0.955
\end{aligned}
$$

"Risk sharing is better than you think" meaning marginal utility growth is very correlated across countires.

2. Asset pricing, "or exchange rates are too smooth." Imposing $\rho = 0$,

$$\sigma(\Delta s_{t+1}) = \sqrt{2}\sigma(m) = 1.41 \times 0.5 = 0.71$$

We should see 70% variation in exchange rates!

3. Consumption. If we use consumption data, Δc, small risk aversion γ, and $\rho = 0$ as suggested by the data, no matter what we do with ρ on the right hand side $\sigma(m)$ is just not enough to add up to the observed $\sigma(\Delta s)$. $\sigma(m) = \gamma\sigma(\Delta c)$.

$$
\begin{aligned}
0.15^2 &= 2 \times \gamma \times \sigma^2(\Delta c) - 2\rho \times \gamma \times \sigma^2(\Delta c) \\
&= 2 \times \gamma \times \sigma^2(\Delta c) \times (1 - \rho) \\
\frac{0.15}{\sqrt{2}} &= 0.106 = \gamma\sigma(\Delta c)(1 - \rho)
\end{aligned}
$$

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This isn't as bad as the equity premium (that's the whole point), where we needed $\gamma\sigma(\Delta c) = 0.5$. It's still not easy. With $\rho = 0$ and $\sigma(\Delta c) = 0.01$ we need $\gamma = 10$. $\sigma(\Delta c) = 2\%$ gets us down to $\gamma = 5$. And surely there's some positive correlation. In sum, you can see that models (especially models with risk sharing) have trouble producing *enough* exchage rate movement.

The paper details:

- We compute a "risk sharing index"

$$1 - \frac{\sigma^2(\ln m^f - \ln m^d)}{\sigma^2(\ln m^f) + \sigma^2(m^d)} = 1 - \frac{\sigma^2(\ln e_{t+1}/e_t)}{\sigma^2(\ln m^f) + \sigma^2(m^d)}$$

Why? $\ln m^f = 2\ln m^d$ also violates risk sharing, but *correlation* is one.

- Procedure: Just like Hansen-Jagannathan. Find the minimum variance discount factors m to price both domestic and foreign assets, expressed in dollars, and vice versa. We use continuous time so we can do logs vs. levels ("E(log) = log(E) theorem" is true in continuous time, with $1/2\sigma^2$ terms)

- Continuous time

$$\Lambda^d = e\Lambda^f$$
$$d\ln\Lambda^d = d\ln e + d\ln\Lambda^f$$

$$\frac{dS}{S} = (r + \mu)\,dt - \sigma dB$$
$$\frac{d\Lambda}{\Lambda} = -rdt - \mu'\Sigma^{-1}\sigma dB$$
$$\frac{d\ln\Lambda}{\Lambda} = -\left[r + \frac{1}{2}\mu'\Sigma^{-1}\mu\right]dt - \mu'\Sigma^{-1}\sigma dB$$
$$\sigma^2\left(\frac{d\ln\Lambda}{\Lambda}\right) = \mu'\Sigma^{-1}\mu$$

so the regular calculation works in logs in continuous time. **Table 2,3** gives the basic calculation.

Strongly recommended reading:

- The introduction on transport costs (Earth vs. Mars) and incomplete markets, and "reconciliation" p. 692 i.e. apples and oranges.

- *Earth vs. Mars*: Just read 673. Better in print. Suppose there are complete financial assets and communication but no goods may flow. If mars gets a good shock, mars stock goes up. The exchange rate must go down. m^f and m^d must be uncorrelated in the end. Knowing this, there is no advantage to Mars stock in the first place, so your portfolio should be completely home biased. *Complete financial assets do not imply "perfect risk sharing" nor constant exchange rates, nor absence of home bias.* Money, capital can't "flow". International is about transport costs, not markets. (Example, crashes "investors pulled capital out." They can't They can sell to locals at a cheap price; trade claim on US govt for claim to factory, but it needs a ship to remove capital.)

- *Incomplete markets.* Now $m^i = m^* + \varepsilon^i$ again.

  1. Reminder: Risk sharing in incomplete markets:

  $$\begin{aligned} p &= E(m^i x) = E\left[proj\left(m^i|x\right)x\right] = E(m^* x), \\ m^i &= m^* + \varepsilon^i \end{aligned}$$

  we should "use asset markets to share as much as possible."

  2. It is *not* true that $m^d = m^f + \Delta s$ for arbitrary m pairs. It *is* true that for any m^d that prices assets expressed in dollars, we can construct *an* m^f that prices assets expressed in Euros. Again, this is just a change of units. But that discount factor may not equal foreign consumption growth. It *is* true that $m^{*d} = m^{*f} + \Delta s$. Thus *minimum-variance discount factors in the payoff space do obey the identity.* In this sense, what we are learning is "do transport costs mean that we are not able to use asset markets to share as many risks as possible?"

- The paper also asks if incomplete markets are quantitatively plausible. $\sigma(m)$ rises as m becomes less correlated, so incomplete markets makes the equity premium puzzle that much worse (as in the "correlation puzzle" refinement of Hansen-Jagannathan that lower correlation of m with r implies a higher $\sigma(m)$ bound).

- How much international finance uses vs. does not use complete markets?? Be very careful here. if

$$e = m^d - m^f$$

and then

$$e = \gamma \Delta c_{t+1}^d - \gamma \Delta c_{t+1}^f$$

you *are* assuming complete markets. If

$$e = proj(\Delta c|assets) - proj(\Delta c^f|assets)$$

then you're not.

Misconceptions:

1. *Each investor is allowed to invest in all assets* – the HJ, minimum variance discount factor for all assets as viewed by each investor. Our equation only applies as a change of units. These are NOT the minimum variance discount factor for domestic assets and the minimum discount factor for foreign assets. Why not? You can compute such quantities, but they are not connected by the exchange rate.

   The paper does *nothing* about "what if there are asset market frictions so you can't trade each other's assets?" We allow markets to be incomplete, but once D can buy them, so can F. Would it be interesting to give the countries fundamentally different spaces, or (the same thing) prices that are different by shadow costs or transactions costs as well as by exchange rates? Yes, but we didn't do it.

2. The "discount factor" is *not* the "optimal portfolio" the "market portfolio", or a portfolio anyone actually holds. The *correlation* of discount factors means *nothing* about correlation of portfolios. For example, if our income shocks are the same, we can have very correlated discount factors, very correlated m^* but utterly different portfolios.

3. The correlation of stock markets, which underlie the usual "benefits of international diversification," is not really in the calculation at all. (Technically, it is reflected in the Σ part of $\mu'\Sigma^{-1}\mu$, but higher asset correlation *does not imply* higher discount factor correlation.) Again, we allow trade in both assets by both investors.

4. Converesly, the discount factor is not a portfolio that anyone holds, so highly correlated discount factors do not mean portfolios are highly correlated. Thus, also, home bias does not contradict our calculations.