Production-Based Measures of Risk for Asset Pricing*

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Abstract

A stochastic discount factor for asset returns is recovered from equilibrium marginal rates of transformation of output across states of nature, inferred from the producers' first order conditions. The marginal rate of transformation implies a novel macro-factor asset pricing model that does a reasonable job explaining the cross section of stock returns with plausible parameter values. Using a flexible representation of the firms' production technology, the producers' ability to transform output across states of nature is estimated to be high, in contrast with what is typically assumed in standard aggregate representations of the firms' production technology.

JEL Classification: E23, E44, G1, G12

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1 Introduction

The goal of this paper is to do for the production side the exact analog of the consumption-based asset pricing paradigm. In a consumption-based approach we use the consumers' first order conditions to recover a stochastic discount factor for asset returns from marginal rates of substitution and with no information about the firms' production technology. In this paper, I use the producers' first order conditions to recover a stochastic discount factor for asset returns from marginal rates of transformation and with no information about preferences. I develop a novel procedure to measure the marginal rates of transformation in practice from industry output and price data. This procedure implies a novel macro-factor asset pricing model that does a reasonable job explaining the observed cross-sectional variation in stock returns with plausible parameter values of the firms' production technology.

As we need a utility function to measure marginal rates of substitution from consumption data, in this approach we need to specify a production function to measure marginal rates of transformation from production data. The left panel of Figure 1 illustrates the economics behind this approach. We observe the producer's choice of date or state-contingent output (point A). The specification of a production function determines the production possibilities frontier across states of nature, represented here by the black solid line. We can then calculate the stochastic discount factor (M_t) that must have led to the observed output choice from the marginal rate of transformation, measured by the derivative of the production possibilities frontier at the production point. The marginal rate of transformation thus identifies a valid stochastic discount factor and thus can be used to price any asset in the economy.

[Insert Figure 1 here]

The challenge in this production-based approach to asset pricing is that standard aggregate representations of the firms' production technology cannot be used directly since they do not have well defined marginal rates of transformation across states of nature. To understand this limitation, consider a typical production function of the form

$$Y(s) = \epsilon(s)F(K) \tag{1}$$

in which K is an input (chosen today), $Y(s)$ is the output and $\epsilon(s)$ is an exogenous productivity level, which are a function of tomorrow's state of nature s. The producer can only increase output in one state of nature tomorrow by increasing the use of the input K, but this will increase production in *all* the other states as well. The corresponding production possibilities frontier is thus Leontief across states, as represented by the black solid line in the right panel of Figure 1. Because of the kink in the production possibilities frontier, the marginal rate of transformation across states is not well defined.[1]

To address this issue, I consider a flexible representation of the firms' production technology that is smooth across states of nature, and nests standard representations as a special case. In this representation, the producer has access to a standard aggregate technology such as in equation (1) but is allowed to choose the state-contingent productivity level $\epsilon(s)$ subject to a constraint set, in order to produce more in high-value states at the expense of producing less in low-value states. The corresponding production possibilities frontier is thus smooth with well defined marginal rates of transformation across states of nature, as illustrated in the left panel of Figure 1. This representation of the firms' technology was first proposed in Cochrane's (1993) note on production under uncertainty but its empirical implications for asset pricing have not been studied before. I discuss why a smooth representation of the firm's technology is a reasonable description of the firms' production process in Section 2.1 below.

To formally establish the equivalence between the marginal rate of transformation and the stochastic discount factor in the economy, I consider the production decision problem of a producer that has access to the smooth production technology discussed above and chooses its inputs to maximize the firm's market value. As I show below (section 2.3), the producer's first order conditions with respect to the state-contingent productivity level, and hence for the state-contingent output Y_t, can be approximately written as

$$M_t = \left(\frac{P_{t-1}}{P_t}\right)\left(\frac{Y_t}{Y_{t-1}}\right)^{\alpha-1}\left(\frac{\Theta_t}{\Theta_{t-1}}\right)^{-\alpha}, \tag{2}$$

where α is a parameter of the firm's production technology, P_t is the price of the firm's output, and $\Theta_t > 0$ is a state-contingent technological parameter that describes the ability of the firm to produce in each state of nature (section 2.1 presents the production technology in detail). These first order conditions say that a value maximizer producer equates the market determined stochastic discount factor M_t to the marginal rate of transformation state-by-state. Thus these conditions allows us to recover a stochastic discount factor from the producers' first order conditions without any information about preferences, in strict analogy to the consumption-based approach to asset pricing.

The marginal rate of transformation in equation (2) depends on an unobserved state-contingent technological parameter Θ_t and thus its implications for asset pricing cannot be examined directly in the data. To solve this identification problem, I extend Cochrane's (1993) framework from an economy with only one aggregate technology to an economy with

[1] This result also holds in more general representations of the technology in which some inputs such as capital or labor utilization are allowed to be adjusted after the state of nature is realized. Naturally, once a state of nature is realized, no transformation of output across states is possible by definition. I discuss this issue in section 2.3.

an arbitrary number of technologies (sectors) and I explore the hypothesis that the state-contingent technological parameter Θ_t is related across technologies. This extension provides cross-equation restrictions linking the first order conditions of the different technologies in the economy in each state of nature. As I show below (section 2.4), these restrictions allows me to recover the unobserved state-contingent technological parameter Θ_t, and hence the equilibrium marginal of transformation, from the *relative* movements in observed price and output data in the different technologies. As an application of this procedure, I consider a two technologies representation of the US economy, which I identify in the data as the durable goods and the nondurable goods sectors, and I show that the equilibrium marginal rate of transformation can be expressed in terms of observable variables as

$$M_t = \exp\left[-b^p\left(\Delta p_{NDt} - \Delta p_{Dt}\right) - b^y\left(\Delta y_{NDt} - \Delta y_{Dt}\right)\right], \tag{3}$$

where Δp_{it} and Δy_{it} are, respectively, the growth rate of prices and output in the nondurable goods (ND) and durable goods (D) sectors, and b^p and b^y are the factor risk prices, which are a function of the parameters of the firms' production technologies in the two sectors. Thus a novel macro-factor asset pricing model follows from a production-based asset pricing setup.

If the marginal rate of transformation is a valid stochastic discount factor, observable cross-sectional differences in average stock returns should be explained by cross-sectional differences in the assets' return covariances with the marginal rate of transformation in equation (3). I test this prediction in the data. I show that the marginal rate of transformation captures reasonably well the risk and return trade-off of several portfolio sorts, including the size and value premia as well as the premia in risk sorted and industry portfolios. The estimated parameters of the firms' production technology are similar across the different test assets which increases confidence in the robustness of the results. In addition, the performance of the model compares well with that from the Yogo (2006) durable consumption based model, a successful representative of the consumption-based approach to asset pricing. As an extension of the two technologies benchmark model, I also consider a multi-technology economy in which case the marginal rate of transformation is identified from output and price data from a larger cross section of sectors. I show that the empirical results are similar to the two technologies representation of the economy suggesting that the empirical performance of the model is robust to the particular choice of the durable versus nondurable sectors in the benchmark model.

To assess the plausibility of the parameter estimates, I perform several diagnostics. First, the marginal rate of transformation estimated in the data implies a stochastic discount factor with sensible properties, in particular, its quite volatile and countercyclical. Second, as an out sample test, the estimated marginal rate of transformation captures the spread in the returns of some portfolios not included in the estimation of the model. The model captures reasonably well the spread in the returns of bond portfolios, but it does not capture the spread in the return of the currency portfolios (Lustig and Verdelhan, 2006). Finally, the fitted time-series dynamics of the estimated marginal rate of transformation shows that the production-based model is able to successfully capture the time variation and volatility of the equity premium and conditional market Sharpe ratio about as well as the aggregate dividend-yield.

The empirical results support the hypothesis that aggregate production technologies are smooth not only across time but also across states of nature. The parameter estimates obtained here suggests that the producers' ability to transform output across states of nature is high in contrast with what is typically assumed in standard aggregate representations of the firms' production technology. Without any ability to transform output across states of nature, I estimate that the observed volatility of the productivity level (aka Solow residual) in the economy should have been about 89% per annum. In the data however, typical values of the volatility of the productivity level are an order of magnitude below, suggesting that firms are substantially smoothing their productivity level (and hence output) across states of nature.

1.1 Related Literature

This paper is related to the strand of the asset pricing literature that establishes a link between the production side of the economy and asset prices that is independent of preferences. General equilibrium models depends on a mixture of preference and technological parameters so it is often difficult to understand the relationship between asset prices and the specific properties of preferences and production technologies. By examining the empirical implications of the producers' first order conditions separately, the approach in this paper provides a tractable framework to understand the relationship between production technologies, production data and asset prices that is robust to misspecification on the consumption side of the economy. Understanding these relationships can also be used to improve the specification of current general equilibrium models. The recent identification of successful utility functions in the consumption-based approach and its subsequent incorporation into general equilibrium models provides support for this view.[2] Cochrane (1991, 2007) and Jermann

[2]For contributions in the consumption-based asset pricing literature (and successfull utility functions) see the review papers by Campbell (2003) and Cochrane (2005). An incomplete list of general equilibrium models with nontrivial consumption and production sectors include: Brock (1982), Rouwenhorst (1995), Jermann (1998), Berk, Green and Naik (1999), Boldrin, Christiano and Fisher (2001), Gomes, Kogan and Zhang (2003), Gourio (2005), Gala (2005), Gomes, Kogan and Yogo (2006), Panageas and Yu (2006) and Papanikolaou (2007).

(2008) provide additional motivations for this approach.

The work most closely related to mine is Cochrane (1988) and Jermann (2008). Using standard spanning arguments, they explore a disaggregated representation of the firms' technology to show that a stochastic discount factor can be uniquely recovered from the producers' first order conditions provided that producers have access to as many technologies as the number of states of nature in the economy. For tractability, they focus on a two-states of nature economy and through simulation, are able to replicate some interesting stylized asset pricing facts such as the equity premium and the term premium. The key feature that differentiates my approach is that the flexible aggregate representation of the firms' production technology that I consider here allows me to read the marginal rate of transformation, and hence the stochastic discount factor, for any number of states of nature in the economy which is naturally important for empirical implementations and testing.

An interesting alternative production-based approach models directly the firms' stock returns in a q-theory framework that also does not require information about preferences. Cochrane (1991) and Rockey and Rockinger (1994) establishes the equivalence between stock returns and investment returns, which can be measured in the data through a production function from investment and output data. Cochrane (1991) confirms that aggregate investment returns are highly correlated with aggregate stock returns. Liu, Whited and Zhang (2007) extends this approach to explain the cross section of equity returns at the portfolio level. Finally, Chen and Zhang (2009) uses this framework to construct production-based factor mimicking portfolios based on firms' characteristics shown to be related to differences in average returns across firms, and finds interesting empirical support in the cross-section. None of these models however, provide a theory for the stochastic discount factor as I do here. Thus, by modeling stock returns directly, this approach provides a characteristics-based explanation of the cross-sectional variation in average stock returns. In contrast, I provide a risk-based explanation of the cross-sectional variation in average stock returns by linking differences in average stock returns to differences in the covariances of the assets' returns with a stochastic discount factor, as measured by the marginal rate of transformation. Naturally, both approaches are complementary and can potentially be combined in future research.

Cochrane (1996), Li, Vassalou and Xing (2006) and Gomes, Yaron and Zhang (2006) explore empirically the hypothesis that aggregate investment returns are factors for asset returns. This hypothesis however, is not a direct theoretical prediction from these models, since the standard production functions in these studies do not have well defined marginal rates of transformation across states. As a result, the factor risk prices in these models are not constrained by the theory and thus are estimated as free parameters, which limits the economic interpretation of the empirical findings. As emphasized by Lewellen, Shanken and Nagel (2006) and many others, the possibility of evaluating the economic magnitude of the factor risk prices by relating them to interpretable properties of the firms' technology or consumers' preferences is an important diagnostic in the evaluation of any asset pricing model.

Finally, Balvers and Huang (2006) links a stochastic discount factor to production variables in a neoclassical economy and also finds empirical support in the cross-section of stock returns. Balvers and Huang's work is not independent of preferences however. The model rules out preference side features such as durable goods, habit formation, long-run risk, preference shocks, etc. In addition, a stochastic discount factor is recovered because of the ability of the consumers, not of the producers, to substitute consumption across states of nature, since the standard neoclassical production function considered by Balvers and Huang does not have well defined marginal rates of transformation across states of nature. As a result, the factor risk prices are not restricted by the theory and thus are estimated as free parameters which, as discussed above, limits the economic interpretation of the results and the scope of the approach.

The paper proceeds as follows. Section 2 presents the production-based asset pricing model. Section 3 discusses the asset pricing implication of the production-based model for the cross-section of stock returns, riskfree rate and time varying risk premia. Section 4 presents the data used, discusses two alternative empirical specifications of the model and the estimation methodology. Section 5 tests the production-based model on the cross-section of stock returns of several portfolio sorts. Section 6 interprets the empirical results. Finally, Section 7 concludes.

2 A Production-Based Model

I present the aggregate flexible representation of the firms' production technology that is smooth (differentiable) across states of nature and consider the optimal production decision problem of a producer in the economy. I derive the link between the stochastic discount factor and the marginal rates of transformation using the producer's first order conditions and propose a procedure to measure the marginal rates of transformation in the data.

2.1 Technology

The output Y_t of each producer in the economy is determined by a standard technology of the form

$$Y_t = \epsilon_t F\left(K_t\right) \tag{4}$$

where $F(.)$ is an increasing and concave function of the inputs K_t. Following Cochrane (1993), each producer is allowed to choose the state-contingent productivity level ϵ_t subject to the constraint set defined by the following analytically tractable CES aggregator[3]

$$\mathbb{E}\left[\left(\frac{\epsilon_t}{\Theta_t}\right)^{\alpha}\right]^{\frac{1}{\alpha}} \leq 1, \tag{5}$$

where $\alpha > 1$ is a parameter and $\Theta_t > 0$ is a state-contingent technological parameter that describes the ability of a producer to generate output in each state of nature: it is relatively easy to produce (i.e. choose high productivity levels ϵ_t) in states with high Θ_t and vice-versa. This representation has sensible properties. The restriction $\alpha > 1$ guarantees that the production possibilities frontier is strictly concave (and thus smooth) across states of nature. In order to increase output in one state of nature the producer must decrease output in the other states and at an increasing rate. This property reflects realistic diminishing returns to scale in the production of output in each state.

In this specification, the curvature parameter α controls the producer's ability to transform output across states. The case $\alpha \to \infty$ corresponds to the standard representations of the production technology. In this case, the producer has effectively no ability to transform output across states of nature since the chosen productivity level ϵ_t must converge to Θ_t state-by-state in order to satisfy the restriction in equation (5). The choice $\epsilon_t = \Theta_t$ is always feasible and as α decreases, it becomes easier for the producer to transform output across states. Thus this restriction can be interpreted as follows: nature hands the producer an *underlying* state-contingent productivity level Θ_t, which the producer distorts into a new state-contingent productivity level ϵ_t in order to produce more in some states of nature (high value states) at the expense of producing less in other states of nature (low value states). This underlying state-contingent productivity level Θ_t captures exogenous technological constraints inherent to the production of output in each state of nature.

The hypothesis that producers have some control over their state-contingent productivity level, and hence their state-contingent output, is plausible. This hypothesis captures the notion that firms respond to uncertainty by adjusting their production processes. A simple example, adapted from Cochrane (1993), helps to illustrate this adjustment in practice. Consider a farmer that can plant in two fields, but the first field only produces if the state of nature "good weather" occurs, while the second field only produces if the state of nature "bad weather" occurs. If the farmer has a limited amount of seeds to use, the decision of how many seeds to allocate in each field determines the exposure of the farmer's total

[3]Feenstra (2003) proposes a similar transformation curve. However, instead of choosing the output across states, he considers the choice of different output varieties.

output (given by the sum of the output in each field) to the aggregate uncertainty. For example, if the farmer chooses to allocates all the seeds in the first field, then the farmer only generates output if the "good weather" state occurs. The intermediate cases generate a smooth production possibilities frontier across states of nature as I have here. This analysis is consistent with the empirical evidence provided in Sheffi (2005) and with the literature on operational risk management (e.g. Apgar, 2006).

Smooth production sets are also consistent with the literature on production under uncertainty. Chambers and Quiggin (2000) (and references therein) shows that if the different inputs used in the production process are subject to different productivity shocks, the choice of the mix of inputs is equivalent to a state-contingent choice of output. More formally, Cochrane (1993) and Jermann (2008) shows that smooth production sets across states of nature can occur when one aggregates standard production functions that are not smooth. This result is analogous to the standard result that an aggregation of Leontief production functions can produce an aggregate smooth production function such as a Cobb-Douglas. Ultimately however, the reasonability of the hypothesis that producers can transform output across states of nature is an empirical question. By nesting standard representations of the production technology as a special case ($\alpha \to \infty$), the technology that I use here allows me to examine this hypothesis in the data.

2.2 The Producer's Maximization Problem

Each producer in the economy, indexed by the subscript $i = 1, .., N$, is competitive and takes as given both the market-determined stochastic discount factor M_t, measured in units of a numeraire good, and the relative price of its output $P_{it} = p_{it}/p_t$, in which p_{it} is the price of the output and p_t is the price of the numeraire good. Markets are complete and thus the stochastic discount factor is unique and is equal to the contingent-claim price divided by the probability of the corresponding state of nature. The existence of a strictly positive stochastic discount factor is guaranteed by a well-known existence theorem if there are no arbitrage opportunities in the market.[4]

Each producer chooses its inputs in order to maximize the firm's value using the market determined stochastic discount factor to value its cash-flows. The timing of the events is as follows. Output and its price are realized at the end of each period. The producer then chooses the current period investment I_{it-1}, the next period state-contingent productivity level ϵ_{it} and distributes the total realized output minus investment costs as dividends D_{it-1} to the owners of the firm.

[4]See for example, Cochrane (2002), chapter 4.2 and references therein.

To derive the first order conditions, it is useful to state the problem recursively. Define the vector of state variables as $x_{it-1} = (K_{it-1}, \epsilon_{it-1}, P_{it-1}, Z_{it-1})$ where K_{it-1} is the current period stock of capital, ϵ_{it-1} is the current period productivity level and P_{it-1} is the current period relative price of good i. The variable Z_{it-1} summarizes the information about the next period distribution (i.e. state-by-state values and probabilities) of the stochastic discount factor M_t, the underlying productivity level Θ_{it} and the relative price of good i, P_{it}. Let $V(x_{it-1})$ be the present value of the firm at the end of period $t-1$ given the vector of state variables x_{it-1}. The Bellman equation of the producer is

$$V(x_{it-1}) = \max_{\{I_{it-1}, \epsilon_{it}\}} \{D_{it-1} + \mathbb{E}_{t-1}[M_t V(x_{it})]\}$$

subject to the constraints,

$$\begin{aligned} D_{it-1} &= P_{it-1}Y_{it-1} - I_{it-1} && \text{(Dividend)} \\ Y_{it-1} &= \epsilon_{it-1}F^i(K_{it-1}) && \text{(Output)} \\ 1 &\geq \mathbb{E}_{t-1}\left[\left(\frac{\epsilon_{it}}{\Theta_{it}}\right)^{\alpha}\right]^{\frac{1}{\alpha}} && \text{(Productivity Level)} \\ K_{it} &= (1-\delta_i)K_{it-1} + I_{it-1} && \text{(Capital Stock)} \end{aligned}$$

for all dates t. $\mathbb{E}_{t-1}[.]$ is the expectation operator conditional on the firm's information set at the end of period $t-1$, δ_i is the depreciation rate of producer's i capital stock and $F^i(.)$ is the (certain) production function, which is increasing and concave.

2.3 First Order Conditions

The first order condition for the state-contingent productivity level ϵ_{it} is given by (all the algebra is in Appendix A-1)

$$\frac{\epsilon_{it}}{\epsilon_{it-1}} = \phi_{it-1}^{\frac{1}{1-\alpha}} \left(\frac{M_t P_{it}}{P_{it-1}}\right)^{\frac{1}{\alpha-1}} \left(\frac{\Theta_{it}}{\Theta_{it-1}}\right)^{\frac{\alpha}{\alpha-1}}, \tag{6}$$

where

$$\phi_{it-1} = \mathbb{E}_{t-1}[M_t P_{it}/P_{it-1}] / \mathbb{E}_{t-1}\left[(\epsilon_{it}/\epsilon_{it-1})^{\alpha-1}(\Theta_{it}/\Theta_{it-1})^{-\alpha}\right]. \tag{7}$$

Intuitively, condition (6) states that the firms' optimal choice of the productivity level (and hence output) in each state of nature is determined by prices and technological constraints. Naturally, since $\alpha > 1$, the firm chooses a higher productivity level in states of nature in which output is more valuable, i.e. high M_t and P_{it} states, and in states of nature in which it is easier to produce, i.e. high Θ_{it} states. When $\alpha \to \infty$ (as in standard aggregate representations of the technology), equation (6) implies $\epsilon_{it} = \Theta_{it}$ state by state, in which case the realized productivity level does not provide information about the stochastic discount factor in the economy. The variable ϕ_{it-1} in equation (7) is pre-determined at time t and thus, as I discuss below, does not play any role for the empirical analysis in this paper.[5]

We can invert the first order condition (6) to recover the stochastic discount factor from the producer's optimal choice of the productivity level. Rearranging terms, we have

$$M_t = \phi_{it-1} \left(\frac{P_{it-1}}{P_{it}} \right) \left(\frac{\epsilon_{it}}{\epsilon_{it-1}} \right)^{\alpha-1} \left(\frac{\Theta_{it}}{\Theta_{it-1}} \right)^{-\alpha}. \tag{8}$$

This condition states that in order to maximize the firm's value, the producer equates the stochastic discount factor M_t to the marginal rate of transformation state-by-state. Thus with this condition we can recover the stochastic discount factor from the producer's decisions without any information about preferences, in strict analogy to the consumption-based approach to asset pricing. Equation (8) is the main prediction from the theoretical model that I explore in the empirical work.

For empirical purposes, it is convenient to express the stochastic discount factor in terms of directly observed variables, up to the underlying productivity level Θ_{it} which I discuss below. Using the fact that the observed output is given by $Y_{it} = \epsilon_{it} F^i(K_{it})$ and that $F^i(K_{it})$ is pre-determined at time t, we can express the stochastic discount factor in equation (8) as

$$M_t = \bar{\phi}_{it-1} \left(\frac{P_{it-1}}{P_{it}} \right) \left(\frac{Y_{it}}{Y_{it-1}} \right)^{\alpha-1} \left(\frac{\Theta_{it}}{\Theta_{it-1}} \right)^{-\alpha} \tag{9}$$

where $\bar{\phi}_{it-1}$ is again a variable pre-determined at time t. Representing the stochastic discount factor in terms of output instead of the productivity level ϵ_{it} simplifies the empirical implementation of the model. Even though the productivity level ϵ_{it} could be measured in the usual way as a Solow residual, this procedure is subject to possible misspecification errors in the functional form of the production function $F(.)$, as discussed in Burnside, Eichenbaum and Rebelo (1996), for example.

The first order condition for physical investment is given by

$$\mathbb{E}_{t-1}[M_t R^I_{it}] = 1, \tag{10}$$

[5]The variable ϕ_{it-1} only affects the mean of the stochastic discount factor and thus it does not have implications for equity premia (excess returns), that I explore in this paper.

where

$$R_{it}^{I} = (1 - \delta_i) + P_{it}\epsilon_{it}F_k^i(K_{it}) \quad (11)$$

is the (stochastic) investment return. This is the standard condition that the investment return is correctly priced. According to this condition, the firm removes arbitrage opportunities from the physical investment and whatever assets the firm has access to. Naturally, because M_t is a valid discount factor, equation (10) holds for all assets in the economy.

In this paper I abstract from capital adjustment costs and the choice of labor inputs by the firm. These are interesting generalizations of the model that I don't pursue here in order to keep the model simple and transparent and to emphasize the role of the choice of the state contingent productivity level in the across-states predictions in equation (8) that I explore. It is important to emphasize however, that allowing labor (or any other input) to adjust ex post is *not* a substitute for the mechanism to transform output across states of nature that I consider here. To measure a marginal rate of transformation across states of nature it is necessary to have a decision in which more in one state of nature costs less in another state. The mere option to adjust something ex post does not tell us anything about the rate at which a producer give up one thing in one state to get it in another. Thus ex-post decisions are not informative about the underlying stochastic discount factor in the economy.[6]

2.4 Identification

In order to use the marginal rate of transformation in equation (9) as a stochastic discount factor in the data, we need to measure the unobserved underlying productivity level Θ_{it}. The simplest approach would be to assume Θ_{it} =constant and estimate it as an additional parameter. Unfortunately, this approach cannot work in practice. With a constant underlying productivity level, the first order conditions (6) implies that the firm chooses an higher productivity level, and hence a higher level of output, in states of nature with high values of the stochastic discount factor. However, it is well known that states with high values of the stochastic discount factor are associated with less output (recessions), not more. Thus, in order to match the real world, it must be true that the underlying productivity level does vary across states of nature and it is sufficiently low (so that output is also low) when the stochastic discount factor is high. This observation is not an assumption about the stochas-

[6]To show this more formally, consider a two-period setup with only ex-post adjustments in labor. The firm chooses the state-contingent labor inputs (L_{t+1}) to maximize its market value:

$$\max_{L_{t+1}} \mathbb{E}\left[M_{t+1}\left(\epsilon_{t+1}F(L_{t+1}) - w_{t+1}L_{t+1}\right)\right]$$

The first order conditions for the state-contingent labor are $\epsilon_{t+1}F'(L_{t+1}) = w_{t+1}$ state-by-state, where w_{t+1} is the wage rate. This condition does not provide any information about the stochastic discount factor M_t.

tic discount factor and follows naturally from a general equilibrium argument: consumers who eat the output would place an higher value for the stochastic discount factor in states of nature with low output.

To solve the identification problem, I assume a factor structure for the unobserved underlying productivity level as stated in Assumption 1. This assumption imposes a strong restriction on the model thus providing testable empirical content to it.

Assumption 1 (Identification) *The growth rate of the unobserved underlying productivity level in each technology ($\Delta\theta_{it}$) has the following factor structure*

$$\alpha\Delta\theta_{it} = \sum_{j=1}^{J} \lambda_{ij} F_t^j \ \textit{for } i = 1, .., N$$

in which F_t^j is the $j = 1, .., J$ common (across technologies) productivity factor and λ_{ij} is the loading of the underlying productivity level of technology i on the common productivity factor j. The loadings for technology 1 are normalized to $\lambda_{1j} = 1$, $\forall j$.

This assumption is motivated by the well documented existence of common factors in production technologies, a proposition that dates back at least to Burns and Mitchell (1946). Empirically, Sargent and Sims (1977)), Stock and Watson (1989, 2002), Singleton (1980) and Forni and Reichlin (1998) provide support for this proposition by showing that a small number of common factors (typically less than three factors) can track a very large number of economic variables. In addition, it is well known that some technologies are more cyclically-sensitive than others. This property is captured here by the loadings λ_{ij} of the firms' underlying productivity level on the common factors, which are allowed to vary across firms.

The assumption of a linear factor structure with no technology specific idiosyncratic term is naturally a strong assumption. The linear structure can be interpreted as a first order linear approximation of a non linear relationship. More importantly, the absence of a technology specific idiosyncratic term is guided by the empirical implementation of the model. In the empirical section, I identify a technology using aggregated industry level data, not firm level data. At the industry level of aggregation it is more reasonable to assume that the idiosyncratic term is averaged out across firms within the industry. Ultimately however, whether this assumption is plausible or not is an empirical question that I address in the empirical section.

Technically, Assumption 1 imposes a cross-equation restriction between the producers' first order conditions. In turn, this additional restriction allows me to infer the underlying productivity level Θ_{it} in the data, and hence the equilibrium marginal rate of transformation, from the *relative* movements of observed output and price data in the different technologies. This result is stated in Proposition 1.

Proposition 1 *Under Assumption 1 and with $J \geq 1$ common productivity factors, the equilibrium marginal rate of transformation can be identified from output and price data in $J+1$ technologies. The marginal rate of transformation is approximately given by*

$$M_t \approx \kappa_{t-1} \exp\left[-\sum_{i=2}^{J+1} \left[b_i^p \left(\Delta p_{it} - \Delta p_{1t}\right) + b_i^y \left(\Delta y_{it} - \Delta y_{1t}\right)\right]\right] \tag{12}$$

where Δp_{it} and Δy_{it} are, respectively, the growth rate in the price and in the output of technology i's good, κ_{t-1} is a variable pre-determined at time t and the factor risk prices b_i^p and b_i^y are a function of the curvature parameter α and the loadings λ_{ij} of the individual production technologies on the common productivity factors (Appendix A-2 provides the general formula). For the one common productivity factor case ($J = 1$), the two factor risk prices can be written as

$$\begin{bmatrix} b^p \\ b^y \end{bmatrix} = \begin{bmatrix} 1/(1-\lambda) \\ (\alpha-1)/(\lambda-1) \end{bmatrix} \tag{13}$$

where, to simplify notation, $\lambda_{21} = \lambda$. For identification, it is also required that $\lambda \neq 1$.

Proof. *See Appendix A-2.* ■

This Proposition shows that a macro-factor asset pricing model follows from a production-based asset pricing setup. Although the exact specification of the marginal rate of transformation in Proposition 1 is new, its specification is closely related to other empirical popular macro-factor asset pricing models such as the Cochrane (1996) and the Li, Vassalou and Xing (2006) investment-based models. These models use the investment growth rate in several technologies as the pricing factors whereas I use output and price growth rates. To the extent that investment growth rates and output growth are highly correlated within technologies, the production-based model can thus be used to understand some of the puzzling empirical findings of these models. For example, Li, Vassalou and Xing (2006) and Cochrane (1996) find that the factor risk prices in their models have typically opposing signs across technologies. Cochrane (1996, table 9) obtains this result when residential and non-residential investment growth are used as pricing factors. The estimated pattern of the risk prices is not explained in these models since the factor risk prices are free parameters not restricted by theory. Using the production-based model to interpret these findings, we see that the opposing pattern in the sign of the investment (output) growth factors is consistent with the theory in this paper. In an economy with two technologies (as in Cochrane, 1996), the factor prices for the output growth factors in (12) are b^y in technology 2 and $-b^y$ in technology 1.

3 Asset Pricing Implications

This section discusses the asset pricing implications of the production-based model for the cross-section of expected excess stock returns, for the riskfree rate and for the variation in the conditional equity premium and Sharpe ratio of the aggregate stock market.

3.1 Risk Premia

The marginal rate of transformation in equation (12) is a valid stochastic discount factor and thus it has implications for the prices and returns of all assets in the economy, including equity, bonds, derivatives as well as the term structure of interest rates. In this paper, I focus on risk premia and study the implications of the model for the cross-section of *excess* stock returns. This allows me to consider a simplified version of the marginal rate of transformation defined in Proposition 1 that is easy to implement in practice. Since for a vector of excess returns (R_t^e) of tradable assets any valid discount factor M_t satisfies

$$\mathbb{E}_{t-1}\left[M_t R_t^e\right] = 0, \tag{14}$$

we can divide the pre-determined component κ_{t-1} of the equilibrium marginal rate of transformation defined in Proposition 1 into the zero on the RHS of this equation. Equivalently, we can set $\kappa_{t-1} = 1$ without changing the pricing errors of the model. This implies that an alternative valid stochastic discount factor for excess returns is given by

$$M_t = \exp\left[-\sum_{i=2}^{J+1}\left[b_i^p\left(\Delta p_{it} - \Delta p_{1t}\right) + b_i^y\left(\Delta y_{it} - \Delta y_{1t}\right)\right]\right] \tag{15}$$

where the factor risk prices $b's$ are specified in Proposition 1. This discount factor is *proportional* to the true marginal rate of transformation in the model: it measures the component of the marginal rate of transformation that varies across states of nature and therefore has pricing implications for excess returns (risk premia).

To make the asset pricing implications for the cross-section of excess stock returns transparent, we can re-write the asset pricing equation (14) as[7]

$$\mathbb{E}_{t-1}[R_t^e] = -\frac{\mathrm{Cov}_{t-1}(M_t, R_t^e)}{\mathbb{E}_{t-1}[M_t]}. \tag{16}$$

This standard pricing equation tells us that cross sectional variation in average stock returns is explained by cross-sectional variation in the level of risk. The main proposition of the production-based model is that the level of risk of any tradable asset can be measured by the covariance of the assets' returns with the marginal rate of transformation in equation (15). An asset is considered risky if it delivers low returns when the marginal rate of transformation is high ("bad times"). Thus this asset must offer an high risk premium in equilibrium as a compensation for its level of risk.

3.2 Riskfree Rate

The riskfree rate is given by $R_{t-1}^f = \mathbb{E}_{t-1}[M_t]^{-1}$. In order to study the implications of the production-based model for the riskfree rate it would be necessary to solve for the *level* of the marginal rate of transformation. This is more complicated than the analysis in the previous section. As shown in Appendix A-2, equations (10) and (19), the pre-determined component κ_{t-1} in equation (12) is a function of conditional moments of the M_t , Θ_{it} and P_{it} joint distribution. By focusing on risk premia in the empirical work, I avoid any additional assumption on the joint distribution of these variables thus helping to make the results more robust to misspecifications.

It is nevertheless important to investigate if the production-based model *can* be consistent with well known properties of the riskfree rate, in particular, the low mean, low volatility and high autocorrelation while at the same time capture the size of the equity premium in the data. In this section, I show that the flexibility of the general production function that I consider here and the absence of capital adjustment costs suggests that the production-based model can, in principle, simultaneously match these properties.

The investment first order condition in equation (10) implies that the mean of the stochastic discount factor inherits the properties of the productivity level (ϵ_{it}), the relative price level (P_{it}) and the (certain) marginal product of capital ($F_k(K_t)$). Because these variables are relatively smooth in the data, especially at the aggregate level or industry level that I

[7] To obtain this result apply the standard decomposition $\mathbb{E}_{t-1}[M_t R_t^e] = \mathbb{E}_{t-1}[M_t]\,\mathbb{E}_{t-1}[R_t^e] + \mathrm{Cov}_{t-1}(M, R^e)$ to the asset pricing equation (14).

consider here, the mean of the stochastic discount factor, and thus the riskfree rate, is likely to be smooth as well. Although this is a general observation, it is useful to illustrate this claim with an example. Consider the optimal production decision problem of an aggregate representative firm, in which case the relative price is $P_{it} = 1$, assume a constant covariance between the stochastic discount factor and the productivity level ϵ_{it}, and suppose output is given by an AK technology such as $Y_{it} = \epsilon_{it} A K_{it}$. Taken the choice of the state contingent productivity level as given, the investment first order condition (10) implies that the (gross) riskfree is given by[8]

$$R^f_{t-1} = (1-\delta) + \left[\mathbb{E}_{t-1}[\epsilon_t] + Cov\left(M_t, \epsilon_t\right)\right] A \tag{17}$$

where A is a parameter of the technology that in principle can be calibrated to match the unconditional mean of the riskfree rate. More importantly, equation (17) makes clear that the riskfree rate inherits the properties of the aggregate productivity level, which is known to have a low volatility and high persistence. At the same time, the choice of a smooth productivity level by the firm does not imply, from equation (8), that the stochastic discount factor in the economy must be smooth as well, which would create problems for the model's ability to capture the equity premium. If the underlying productivity level Θ_{it} is volatile and strongly negatively correlated with the stochastic discount factor (which I show to be the case in the data), equation (8) is simultaneously consistent with a volatile stochastic discount factor and firms choosing smooth state contingent productivity levels.

The absence of capital adjustment costs is important for this analysis. With capital adjustment costs, the riskfree rate in equation (17) will in general depend on the investment rate.[9] Because the investment rate has some persistence and is quite volatile in the data, the implied riskfree rate would be volatile as well. This excessive volatility of the riskfree rate induced by capital adjustment costs is a common problem in several general equilibrium asset pricing models that are calibrated to match the equity premium with standard technologies, a point first shown in Jermann (1998) and discussed in Cochrane (2007). The general production-function that I consider here can in principle simultaneously match the asset pricing moments of both the riskfree rate and the equity premium because it allows us to separately control the variability of the marginal rate of transformation *across states* of nature, which is determined by the curvature parameter α and by the properties of the

[8]This result follows from equation (10), $\mathbb{E}_{t-1}\left[M_t\left(1-\delta_i+\epsilon_{ti}A_i\right)\right]=1$. Using the standard decomposition $\mathbb{E}_{t-1}[XY] = \mathbb{E}_{t-1}[X]\,\mathbb{E}_{t-1}[Y] + \text{Cov}_{t-1}(X,Y)$, the constant covariance assumption, and the definition of the risk-free rate $\text{R}^f_{t-1} = \mathbb{E}_{t-1}[M_t]^{-1}$ yields equation (17).

[9]Let I_t/K_t be the investment rate. Assuming a standard quadratic specification for the adjustment cost function, output is given by $Y_t = \epsilon_t A K_t - \frac{\beta}{2}\left(I_t/K_t\right)^2 K_t$, and the investment first order condition is given by $\mathbb{E}_{t-1}\left[M_t \frac{\epsilon_t A + \beta (I_t/K_t)^2 + (1+\beta I_t/K_t)(1-\delta)}{1+\beta I_{t-1}/K_{t-1}}\right] = 1$.

underlying productivity level Θ_{it}, from the variability of the marginal rate of transformation *over time*, which is determined by the properties of the certain production function $F(.)$ in equation (4) and possibly by a capital adjustment cost function. The first characteristic allows us to match the equity premium in the data while the second characteristic allows us to match the risk-free rate properties. This separation property of the production technology is similar in spirit to that in the general class of preferences in which the degree of risk aversion and the elasticity of intertemporal substitution can be separately specified (Epstein and Zin, 1989 and 1999).

3.3 Time-Varying Equity Premium and Sharpe Ratio

Under additional assumptions, the production-based model can also be used to address two additional asset pricing facts. The countercyclical equity premium and the volatile and countercyclical Sharpe ratio of the aggregate stock market.[10]

Using the standard pricing equation (16), the conditional equity premium and the conditional Sharpe ratio (SR^M_{t-1}) can be written as

$$\mathbb{E}_{t-1}\left[R^s_t - R^f_{t-1}\right] = -\frac{\sigma_{t-1}(M_t)}{\mathbb{E}_{t-1}(M_t)}\sigma_{t-1}(R^s_t)\,\rho_{t-1}(M_t, R^s_t) \tag{18}$$

$$SR^M_{t-1} = \frac{\sigma_{t-1}(M_t)}{\mathbb{E}_{t-1}(M_t)}\rho_{t-1}(M_t, R^s_t), \tag{19}$$

where R^s_t is the aggregate stock market return, R^f_{t-1} is the risk free rate, and $\sigma_{t-1}(.)$ and $\rho_{t-1}(.)$ are the conditional volatility and the conditional correlation of the relevant variables. Provided that the conditional moments of the marginal rate of transformation and stock returns on the right hand side of equations (18) and (19) can be computed, these equations can be used to obtain predictions for the time varying equity premium and Sharpe ratio in the production-based model and verify if they are consistent with the empirical evidence.

As discussed in Campbell (2003), the time variation in the equity premium is likely to be driven by time variation in the price of risk, which is given by $\sigma_{t-1}(M_t)/E_{t-1}(M_t)$. Interestingly, computing the price of risk in the production-based model does not require solving for the level of the marginal rate of transformation M_t, since the market price of risk does not depend on the unknown pre-determined component κ_{t-1} in Proposition 1, which facilitates the analysis. In order to proceed however, the conditional moments of

[10] For evidence on the time series variation of the equity premium see, for example, Campbell (1987), Campbell and Shiller (1988), Fama and French (1988,1989), and Keim and Stambaugh (1986). For evidence on the time series variation of the conditional Sharpe ratio see, for example, Brandt and Kang (2004) and Ludvigson and Ng (2007).

the component of the marginal rate of transformation that varies across states have to be estimated. In the empirical section, I compute these moments using the fitted values of the marginal rate of transformation (15), and by estimating a time series process for its dynamics.

4 Data, Empirical Specification and Estimation Method

I briefly describe the macro and asset data, discuss two alternative empirical specifications of the model and present the estimation method used to obtain the parameters estimates and test the model.

4.1 Macro Data

I identify a technology in the model as an industry in the US economy, and I use the aggregate industry level data provided in the National Income Product Accounts (NIPA) available through the Bureau of Economic Analysis (BEA) website. NIPA provides industry level data for four aggregate industries: durable goods, nondurable goods, services and structures. I use this dataset because it provides annual data for a long time series from 1930 to 2007, thus helping to increase the power of the statistical tests considered here. Output is measured by the real gross domestic product in each industry, obtained from NIPA table 1.2.3, lines 7, 10, 13 and 14. The price data for each industry is also from NIPA, table 1.2.4, lines 7, 10, 13 and 14. Since the price data for the durable goods and nondurable goods industries is only available after 1946, I use the price data for the sales of durable goods and nondurable goods for the 1930 to 1946 period. This data is from NIPA table 1.2.4, lines 8 and 11.

4.2 Empirical Specification

The theoretical model is silent about the number of common productivity factors in the economy. To establish the robustness of the empirical findings, I estimate and test the production-based model under two alternative empirical specifications that differ in the assumed number of common productivity factors. Both specifications have advantages and disadvantages as I discuss below.

4.2.1 One Common Productivity Factor Specification

The first empirical specification that I consider assumes the existence of only one common productivity factor in the economy. This assumption is reasonable given the empirical findings discussed above (e.g. Sargent and Sims, 1977). With one common productivity factor, Proposition 1 shows that the marginal rate of transformation can be identified from price and output data in two production technologies. Empirically, this specification is appealing since it makes the analysis particularly tractable. Having a small number of pricing factors avoids parameter proliferation and facilitates the economic interpretation of the estimation results. Because of this tractability, this will be the benchmark model in the empirical section.

I interpret the first (reference) technology as the durable goods sector and the second technology as the nondurable goods sector in the US economy. In this case, Proposition 1 and equation (15) implies that the stochastic discount factor is given by

$$M_t = \exp\left[-b^p\left(\Delta p_{NDt} - \Delta p_{Dt}\right) - b^y\left(\Delta y_{NDt} - \Delta y_{Dt}\right)\right] \tag{20}$$

where Δp_{it} and Δy_{it} are respectively the growth rate of prices and output in the i =durable goods (D) and nondurable (ND) goods sector and the factor risk prices are

$$\begin{bmatrix} b^p \\ b^y \end{bmatrix} = \begin{bmatrix} 1/(1-\lambda) \\ (\alpha-1)/(\lambda-1) \end{bmatrix}. \tag{21}$$

From the factor risk prices, we can recover the parameters of the firm's production technology, in particular the curvature parameter α and the loading λ of the nondurable goods sector on the common productivity factor. The estimation of these parameters provides novel information about the characteristics of the firm's production technology and allows us to evaluate if the model can match the data with plausible parameter values.

Naturally, the focus on the durable goods and the nondurable goods sectors is an additional assumption and many other specifications could be considered.[11] This specification is convenient however, since the time series properties of the output growth in these two sectors suggests that Assumption 1 is satisfied by these two sectors. This fact can be seen in the top panel in Table 1 which reports the summary statistics of these variables across the whole sample period and across expansion and recession periods. I define a year as being a recession if there are at least five months in that year that are defined as being a recession

[11] The focus on a small number of sectors is common in the production/investment-based asset pricing literature. Examples include Jermann (2008) who focus on investment in equipment & software vs investment in structures, Cochrane (1996) who focus on residential vs non-residential investment and Cochrane (1988) who focus on consumer durables versus physical capital.

by the NBER. Clearly, output growth in these two sectors has a business cycle component which is consistent with an underlying common factor: not surprisingly, output growth in both sectors tends to be high in expansions and low in recessions. In addition, the two sectors seems to have different sensitivities to the business cycle: output growth in the durable goods sector is on average 14.8% higher in expansions than in recessions while the output growth in the nondurable goods sector is only 2% higher in expansions than in recessions. In turn, this suggests that both sectors load differently on the common productivity factor (i.e. $\lambda \neq 1$). The focus on durable goods and the nondurable goods sectors is also consistent with the approach in Gomes, Kogan and Yogo (2009) who study a similar two-sector decomposition of the US economy to examine a different set of asset pricing questions.

[Insert Table 1 here]

4.2.2 Multiple Common Productivity Factors Specification

The second empirical specification assumes the existence of an arbitrary number of common productivity factors in the economy. The main advantage of this approach is that it allows me to incorporate in the estimation of the model information from a larger cross-section of technologies thus making the results less sensitive to a particular choice of two technologies as in the previous specification. The main disadvantage of this approach is that the underlying parameters of the firm's technology cannot be recovered from the estimated factor risk prices as I show below, which substantially limits the economic interpretation of the results.

Under the assumption of multiple ($J > 1$) common productivity factors, Proposition 1 shows that the marginal rate of transformation can be identified from price and output data in $J+1$ technologies. With a possibly large number of common productivity factors, the use of the marginal rate of transformation in Proposition 1 is not feasible in practice. Output and price growth are highly correlated across sectors (Murphy, Shleifer and Vishny, 1988), which creates multicollinearity problems and makes inference unreliable. In addition, the number of pricing factors, and thus the number of parameters to be estimated, increases with the number of common productivity factors exacerbating the problem. For example, with $J = 3$ common productivity factors the marginal rate of transformation has six pricing factors, namely the relative growth rate of output and relative prices in the three technologies.

To overcome these problems, I reduce the number of pricing factors through a principal components analysis. This analysis summarizes the information contained in the cross section of relative output and relative prices growth in a small number of orthogonal variables - the principal components - that by construction retain most of the information of the original variables. This procedure thus allows me to isolate the components of the cross section of output and relative prices growth that are potentially more relevant for pricing while maintaining tractability. Mardia, Kent and Bibby (1979) provide a textbook treatment of principal components analysis. Appendix A-3 explain the procedure in detail.

For this empirical specification, I consider the cross-section of the four NIPA sectors, namely the durable goods, nondurable goods, services and structures sectors. As in the one common factor productivity specification, I specify the durable goods sector as the reference technology (technology 1). Thus, by using data from four sectors, I'm implicitly assuming $J = 3$ common productivity factors. Focusing on a relatively small number of common factors is advisable given the empirical evidence discussed above that a small number of common factors tracks a large number of economic variables (e.g. Sargent and Sims, 1977). Thus although it would be tempting to consider a much larger cross-section of sectors, that procedure does not seem to be empirically supported.

In order to do a principal components analysis of the cross section of relative price and output growth, it is necessary to first linearize the marginal rate of transformation in (15). Then, as I show in the Appendix A-3, the marginal rate of transformation in this empirical specification can be well approximated by

$$M_t \approx 1 - b^p \text{PFPC}_t - b^y \text{OFPC}_t \tag{22}$$

where PFPC_t (price first principal component) is the first principal component of the cross-section of the relative price growth and OFPC_t (output first principal component) is the first principal component of the cross-section of relative output growth. Each of these first principal components capture 73% (output) and 66% (price) of the cross-sectional variation of the corresponding variable and thus retain a large part of the information of the entire cross-section. In addition, this approximation seems appropriate for asset pricing purposes, as suggested by a series of tests in which only the first principal component of these variables seems relevant for pricing (results not reported here for brevity but available upon request).

With only two pricing factors it is not possible to recover the parameters of interest, in particular the curvature parameter α and all the technology specific loadings λ_{ij} on the common productivity factors, from the two factor risk prices b^p and b^y. As such, the factor risk prices are estimated as free parameters in this empirical specification since they are not constrained by the theory developed here.

4.3 Estimation Methodology

I estimate the parameters of the firm's production technology and test the model by the Generalized Method of Moments (GMM), using the methodology developed by Hansen and Singleton (1982) (see also Cochrane, 2002, for a textbook treatment of GMM). Let z_{t-1} be a vector of instrumental variables known at time $t-1$ and $R^e_{it}(i=1,..,N)$ be a vector of excess returns of N portfolios. From equation (14), the following moment restriction is used for estimation and testing,

$$0=\mathbb{E}[M_t R^e_{it} z_{t-1}]\ ,\ (i=1,..,N). \tag{23}$$

The interaction of the portfolios excess returns with the instruments, $R^e_t z_{t-1}$, can be interpreted as the return on a managed portfolio, in which a manager invests more or less based on the signal provided by z_{t-1} (Cochrane, 1996). The instruments are introduced in order to capture the information that investors have at time $t-1$. In addition to a constant, I use the dividend-price ratio of the aggregate stock market as an instrument since this variable is a known predictor of stock returns (see, for example, Fama and French, 1988). In order to obtain the estimated predicted excess return of each portfolio, I use equation (16). Appendix A-4 provides additional information about the estimation procedure.

In the estimation, I normalize the mean of the marginal rate of transformation to one (since the mean is not identified from the estimation of the model on excess returns). For the one common productivity factor specification of the model, the stochastic discount factor M_t is given by the nonlinear marginal rate of transformation defined in equation (20) and the factor risk prices are given by equation (21). For the multiple common productivity factor specification of the model, the stochastic discount factor M_t is given by the linear approximation of the marginal rate of transformation defined in equation (22) and the factor risk prices are free parameters as discussed above. Because of the linearization of the stochastic discount factor in this specification, some potential pricing information included in nonlinearities is lost. As such, in this specification, I also include a constant (c^{te}) in the moment condition (23), thus allowing the model to missprice the risk free asset, and focus on matching the cross-sectional differences in stock returns. Under the null that the model is correct and that the missing nonlinearities are not important for pricing, the constant should be zero. I test this hypothesis in the data as an additional diagnostic.

4.4 Asset Data

I consider the following test assets in the empirical tests: (i) 6 Fama-French size and book to market portfolios; (ii) 9 risk pre-ranking beta double sorted portfolios; (iii) 5 Gomes, Yogo and Kogan industry portfolios; and (iv) all portfolios together. Appendix A-5 explains the construction of these portfolios in detail.

The choice of these portfolios is justified as follows. First, all these portfolios are available since 1930 thus allowing me to perform the asset pricing tests on a long time series and thus maximize the power of the statistical tests. I use the 6 Fama-French portfolios sorted on size and book to market because these portfolios capture two well known facts that have attracted much attention in the asset pricing literature: the *size premium,* the fact that small stocks (stocks with low market capitalization) have higher average returns than big stocks (stocks with high market capitalization), and the *value premium,* the fact that value stocks (stocks with high book-to-market) have higher average returns than growth stocks (stocks with low book-to-market). I focus on the 6 Fama-French size and book to market portfolios instead of the more standard 25 Fama-French size and book to market portfolios because the information content in the 6 portfolios is almost identical to the information content in the 25 portfolios due to the strong facture structure of these portfolios (the average R^2 of the Fama and French, 1993, three pricing factors (Market, SMB and HML) explains more than 90% of the time-series variation in the returns of these portfolios). In addition, the smaller set of test assets allows me to obtain more reliable estimates. A small number of test assets is recommended since the covariance matrix of the pricing errors is difficult to estimate precisely when a large number of test assets is used which in turn may lead to spurious results and make inference unreliable. I note however that the parameter estimates obtained when the 25 Fama-French portfolios are used as test assets are very similar to the estimates obtained using the 6 Fama-French portfolios (results available upon request).

I also consider other test assets in order to relax the tight factor structure of the size and book to market portfolios and thus address Daniel and Titman (2005) and Lewellen, Nagel and Shanken (2006) critiques. I consider 9 risk pre-ranking beta double sorted portfolios, which are value weighted portfolios that are formed based on the "pre-ranking" relative durable-nondurable price growth and relative durable-nondurable output growth betas of each individual stocks. Sorting on pre-ranking betas provides a rigorous test for asset pricing models by creating a large spread in the post-formation betas or covariances. Table 2 shows that this procedure achieves its goal, despite the natural concern that pre-ranking betas are difficult to estimate precisely and thus are subject to measurement error. The ex-post covariances maintain the monotone relationship and the spread of the ex-ante covariances. In addition, consistent with the hypothesis that these factors are important risk factors, this sorting procedure generates a large spread in average returns (maximum spread of 6.8%) and the relationship between average returns and the corresponding covariances with the factors is monotonic across the two factors. The Patton and Timmermann (2008) monotonic relation test, strongly rejects the hypothesis that the average returns of these portfolios are all equal against the hypothesis that they are decreasing in the output sort and increasing in the price sort, with a p-value of 0.44%.

In addition to risk sorted portfolios, I also examine the Gomes, Kogan and Yogo (2009) 5 industry portfolios since the sorting is based on industry classification which is close in spirit to a production-based approach to asset pricing. Finally, I examine all the portfolios together to test if the parameter estimates are consistent across test assets.

[Insert Table 2 here]

5 The Production-Based Model in Practice

I estimate and test both the one common productivity factor and the multiple common productivity factors specifications of the production-based model on the cross-section of stock returns of several portfolio sorts.

5.1 The Size and Value Premia

I start by examining if the production-based model is able to explain the cross-sectional variation in the average returns of the 6 Fama-French portfolios sorted on size and book to market (6-Size-BM). The first two columns in Table 3, Panel A, reports the first and second stage GMM estimates and tests of the one common productivity factor specification of the production-based model on these portfolios. The model captures well the cross-sectional variation in the returns of these portfolios with an high cross sectional R^2 of 83.9% and low mean absolute pricing errors of 1.4% per annum. More importantly, the $J-$ test of overidentifying restrictions fails to reject the model (p-value of 77.6% in the second stage). Finally, the estimated curvature parameter has the correct theoretical sign ($\alpha > 1$). I interpret the estimates of the technological parameters in Section 6 below.

[Insert Table 3 here]

The top-left panel in Figure 2 provides a visual description of the fit of the production-based model on these portfolios. This figure plots the predicted versus realized excess returns implied by the first stage GMM estimates of the model, excluding the managed portfolios. The straight line is the 45° line, along which all the assets should lie. The deviations from this line are the pricing errors which provides the economic counterpart to the statistical analysis. In the figure, each digit number represents one portfolio. The first digit refers to the size sort (1 and 2 for small and big respectively), and the second digit refers to the book to market sort (1, 2 and 3 indicating the growth, neutral and value portfolios respectively). The six portfolios lie relatively close to the 45° line. Value stocks have higher predicted average returns due to their larger negative covariance with the marginal rate of transformation, thus capturing the *value* premium. Small stocks tend to have a larger negative covariances than big stocks thus capturing the *size* premium. Thus the production-based model suggests that the size and the value premia observed in the data reflect a compensation for risk.

[Insert Figure 2 here]

In the evaluation of any asset-pricing model it is important to understand which facts in the data are responsible for the results. Table 4 reports the covariance of the relative output growth factor (Output) and the relative price growth factor (Price) with the returns of the 6 Fama-French size and book to market portfolios as well as the average returns on these portfolios. There is some, albeit small, spread in the covariances of the output factor with the returns on these portfolios, mainly along the size dimension. There is a large spread in the covariances of the price factor with the returns of these portfolios both along the size and the book-to-market dimensions, and the pattern of the covariances matches that of the average returns reported. Taken together, these results suggests that the price factor is capturing most of the cross-sectional variation in the returns of these portfolios, with the output factor adding some marginal explanatory power along the size dimension.

[Insert Table 4 here]

The results for the multiple common productivity factor specification are reported in the first two columns of Table 3, Panel B. Overall, the results are consistent with the results for the one common productivity factor specification, suggesting that the results for the previous specification are not specific to the particular choice of the durable goods and nondurable goods sectors as the two technologies in the economy. The model is not rejected by the $J-$ test of overidentifying restrictions (p-val of 26.5% in the second stage), it has an high cross sectional R^2 of 84.1% and low mean absolute pricing errors of 1.4% per annum.

5.2 Other Portfolios

Columns three to eight in Table 3, Panel A, report the first and second stage GMM estimates and tests of the one common productivity factor specification of the production-based model on the 9 risk pre-ranking beta double sorted portfolios (9-Risk), the 5 Gomes, Kogan and Yogo (2009) industry portfolios (5-Ind), and all the 20 portfolios together (20-All), including the Fama-French 6 size and book to market portfolios.

Across all test assets, the model is not rejected by the $J-$ test of overidentifying restrictions. In addition, the estimation produces reasonable cross sectional R^2 of 79% and relatively low mean absolute pricing errors of 1.4% per annum, when all portfolios are considered together. Figure 2 plots the predicted versus realized excess returns implied by the first stage GMM estimates of the model on these portfolios, excluding the managed portfolios. For both the 9-Risk and the 20-All portfolios, most of the test assets lie along the 45° line and thus the model generates low pricing errors. In the industry portfolios, the model can capture the relatively large spread between the average returns of the durable goods industries (portfolio 3) and the services goods industries (portfolio 1) documented in Gomes, Kogan and Yogo (2009). The fit of the model on the investment goods producing firms is more modest.

The results reported in Table 3, Panel A also show that the magnitude of the parameter estimates are consistent across all the test assets. This is an important diagnostic in the evaluation of the performance of any asset pricing model since under the null hypothesis that the model is correct, the parameter estimates should be independent of the test assets used.

The results for the multiple common productivity factor specification reported in Table 3, Panel B are largely consistent with all the previous analysis and thus the discussion of its results are omitted here for brevity. I note however, that the constant is statistically significant in the second stage GMM estimation of the model on the 5 industry and all the 20 portfolios together. This result suggests that the linearization of the marginal rate of transformation procedure required for this empirical specification slightly deteriorates the information content of the marginal rate of transformation for asset pricing.

6 Interpreting the Empirical Results

In this section I provide a detailed analysis of the empirical results in order to investigate if the model fits the data with reasonable parameter values and assess the robustness of the results. Here, I focus most of the analysis on the estimation results for the one common productivity factor specification of the production-based model on the Fama-French 6 portfolios sorted on size and book-to-market, since the parameter estimates are similar across test assets.

6.1 Properties of the Firms' Production Technology

The estimates of the technological parameters (α and λ) reported in the first column of Table 3, Panel A reveal interesting information about the characteristics of the technologies in the nondurable and the durable goods sectors. The parameter that controls the sensitivity of the underlying productivity level in the nondurable goods sector to the common productivity factor (parameter λ) is positive but smaller than one. Thus, according to this estimate, the underlying productivity levels in the two sectors are positively correlated but the underlying productivity level in the nondurable goods sector is less sensitive to the common productivity factor. To the extent that the common productivity factor is closely related to the business cycle, this fact helps to explain why the output growth in the nondurable goods sector is less cyclical than the output growth in the durable goods sector as reported in the top panel in Table 1.

The point estimate of the curvature parameter α is small ($\hat{\alpha} = 1.04$) but is greater than one, which is the minimum admissible value for this parameter. This result suggests that producers have some ability to transform output across states in contrast with what is typically assumed in standard aggregate representations of the firms' production technology. Without a formal test available (this parameter is not identified under the null), it is clear that the hypothesis that the producers' have no ability to transform output across states ($\alpha \rightarrow \infty$) is not supported by these estimates.

The curvature parameter α is the production-based analogue of the coefficient of relative risk aversion in the standard consumption-based model. Since this parameter is new in the literature, there is no benchmark to compare this value with. In order to interpret its economic magnitude, I examine the properties of the growth rate in the estimated underlying productivity level (Θ_t) and relate it to well documented time series properties of the growth rate in firm specific realized productivity levels (aka Solow residuals). As discussed in section 2.1, when the firm cannot transform output across states of nature, the underlying and the realized productivity levels should be equalized state by state. Examining the difference between the properties of these two series thus provides an indirect measure of how much transformation of output across states firms are doing.

As shown in equation (17) in Appendix A-2, the common productivity level (and hence the underlying productivity level through identification Assumption 1) can be inferred from price and output data in the two sectors from

$$F_t = (\lambda - 1)^{-1} \left[\gamma_{\mathrm{ND}t-1} - \gamma_{\mathrm{D}t-1}\right] - (\lambda - 1)^{-1} \left[\Delta p_{\mathrm{D}t} - \Delta p_{\mathrm{ND}t} - (\alpha - 1)\left(\Delta y_{\mathrm{ND}t} - \Delta y_{\mathrm{D}t}\right)\right], \tag{24}$$

where $\gamma_{\mathrm{ND}t-1}$ and $\gamma_{\mathrm{D}t-1}$ are pre-determined variables at time t. The difficulty with this equation is that γ_{it-1}, in general, varies over time. Here, I make the simplifying assumption that $\gamma_{\mathrm{ND}t-1}$ and $\gamma_{\mathrm{D}t-1}$ are approximately constant. Note that this assumption is not required for any of the asset pricing tests reported in the previous sections. In addition, because the mean of the common productivity level is not pinned down in the estimation of the model on excess returns, I focus my analysis on the demeaned values of this variable. As reported in the third panel in Table 1, the mean of the estimated common productivity factor F_t is -0.47 in recessions and 0.23 in expansions. This result is sensible since it suggests that on average, recession periods corresponds to the realization of states of nature in which it is difficult to produce (low F_t states). Interestingly, this table also shows that the estimated standard deviation of the common productivity factor is about 89% (full sample). For comparison, typical estimates of firm level volatilities of the realized productivity level reported in Cooper, Russell, and Haltiwanger (2005) are around 22%, and estimates at the industry or at the macro-aggregate level are even lower. This analysis shows that the estimated curvature parameter $\hat{\alpha} = 1.04$ allows firms to substantially smooth their productivity level suggesting that firms have a large ability to transform output across states of nature.

Figure 3, plots the time series of the estimated common productivity factor. The shaded bars are the NBER-recession years. The plot reveals that although the common productivity factor is on average lower in NBER-recessions and higher in NBER-expansions, the pattern is not perfect. For example, the recessions in 1945 and 1973 correspond to the realization of states in which it was easy to produce (high F_t state).

[Insert Figure 3 here]

6.2 Implied Stochastic Discount Factor

It is also interesting to examine the time series of the fitted marginal rate of transformation. This time series naturally provides information about the realized stochastic discount factor in the US economy thus allowing us to assess if the estimated model generates a stochastic discount factor with plausible properties. The estimation of this time series follows directly from equation (18) in Appendix A-2 assuming, similarly to the previous section, that the predetermined variable $\varkappa_{t-1}$ is approximately constant over time.

The third panel in Table 1, shows that the marginal rate of transformation (demeaned and in log) tends to be considerably higher during NBER-recessions than during NBER-expansions, as we should expect from any reasonable asset pricing model. The marginal rate of transformation is 0.49 in recessions and -0.24 in expansions. Figure 3, plots the estimated time series of the marginal rate of transformation. Interestingly, the plot reveals recession states (high M_t) that are not captured by the NBER-designated business cycle recessions dates. In 1956 we observe a large realization in the marginal rate of transformation, but this year is not classified by the NBER as a recession. More importantly, not all NBER-recessions were equally important. According to the production-based model, the recessions in 1930, 1938 and 1949 were particularly severe since they correspond to the realization of states of nature with very high marginal rates of transformation. Conversely, the NBER-recession in 1973 is not reflected in the realization of an high marginal rate of transformation. In short, the estimated marginal rate of transformation captures information about the severity of recessions that transcends the NBER-designated business cycle recessions dates.

Figure 3 also shows that the innovations in the marginal rate of transformation and the innovations in the common productivity factor are almost the mirror image of each other. The correlation between the two innovations is -0.98 (in levels this correlation is smaller, -0.38) and the volatility of the innovations (reported in Table 1) in the common productivity factor (89%) is approximately equal to the volatility in the log marginal rate of transformation (94%). This result is not surprising given the relatively low volatility of output and price growth in the two sectors when compared with the required volatility of any valid discount factor that prices assets in the US economy. To understand this fact, recall that the marginal rate of transformation in the sector 1 (here, the durable goods sector) can be written as

$$M_t = \bar{\phi} \left(\frac{P_{\mathrm{D}t}}{P_{\mathrm{D}t-1}} \right)^{-1} \left(\frac{Y_{\mathrm{D}t}}{Y_{\mathrm{D}t-1}} \right)^{\alpha-1} F_t \ , \tag{25}$$

which follows from equation (9), the identification Assumption 1 and letting $\bar{\phi} = \bar{\phi}_{it-1}$. From Table 1, the standard deviation of annual output growth in the durable goods sector is approximately 13% and the standard deviation in the relative price growth is around 2.6%.[12] Given that the Sharpe ratio in the US economy in the post-war period is approximately 0.4, this implies that the standard deviation of the discount factor must be at least 40% on annual data.[13] To be consistent with these values, and given the low point estimates of the curvature parameter α, equation (25) immediately implies that we need a volatile common productivity factor that is highly negatively correlated with the stochastic discount factor in order to make the model consistent with the observed relatively low volatility of output and (relative) price growth.

6.3 Comparison with the Consumption-Based Approach

In order to evaluate the production-based model, it is also interesting to compare it to other asset pricing models rather than simply reject or fail to reject it on the basis of statistical tests. In fact, it is not hard to statistically reject any of the current popular models if one

[12] Note that, in this equation, P_{Dt} is the *relative* price of the durable good, with respect to the price of the numeraire good. Here, I consider the numeraire good to be the aggregate consumption basket in which case the CPI for all goods is used.

[13] This analysis follows from the basic pricing equation for excess returns (R^e)

$$0 = \mathbb{E}\left[MR^e\right] = \mathbb{E}\left[M\right]\mathbb{E}\left[R^e\right] + \rho\left(M, R^e\right)\sigma\left(M\right)\sigma\left(R^e\right)$$

we have

$$\sigma\left(M\right) = -\frac{\mathbb{E}[M]}{\rho\left(M, R^e\right)}\frac{\mathbb{E}\left[R^e\right]}{\sigma\left(R^e\right)}.$$

The unconditional Sharpe ratio in the US postwar data is about $\mathbb{E}\left[R^e\right]/\sigma\left(R^e\right) = 0.4$ annualy. Thus, even if the discount factor and returns are perfectly correlated ($\rho\left(M, R^e\right) = 1$) we need $\sigma\left(M\right) = 40\%$ annually.

uses a sufficiently rich set of test assets or a data sample covering a sufficiently long period.

In this section, I compare the production-based model with the Yogo (2006) durable consumption asset pricing model. This model is an extension of the Lucas (1978) and the Breeden (1979) standard consumption-based model with power utility. Yogo (2006) consider a more general specification of preferences that are nonseparable between durable and nondurable consumption goods and have a recursive representation following Epstein and Zin (1991). Here, I compare my estimation results for the Yogo (2006) model with those reported in Gomes, Kogan and Yogo (2009) (henceforth GKY). GKY re-estimate the Yogo (2006) model on annual data and for the same sample period used in the estimation of the production-based model, 1930 to 2007, thus allowing for a more direct comparison (Yogo, 2006, is estimated on quarterly data and for the 1951 to 2001 period).

Yogo (2006) durable consumption model is an appropriate benchmark for several reasons. First, this model is representative of the consumption-based approach to asset pricing and thus it is a natural theoretical benchmark for the production-based model. Second, Yogo (2006) and Lustig and Verdelhan (2007) show that this model can successfully explain the cross-sectional variation in the returns of several test assets in the post 1951 period. Third, the focus on durable and nondurable consumption is close in spirit to the decomposition across durable and nondurable goods producing firms that I explore here. Fourth, the data to estimate the model is available since 1930, which is the starting period of the production data that I use here. Finally, the durable consumption model is nonlinear and the pricing factors are based on macroeconomic variables, thus allowing for a fair comparison. Linear portfolio-based models such as Fama and French (1993) or Chen and Zhang (2009) use better measured and more frequent data, so they tend to work better in sample, in comparison with macro-asset pricing models.

In the consumption-based approach to asset pricing, the stochastic discount factor in the economy (M_{t-1}) is recovered from the equilibrium intertemporal marginal rate of substitution (IMRS). In Yogo (2006), the IMRS is given by

$$M_{t-1} = \left[\beta \left(\frac{C_t}{C_{t-1}}\right)^{-1/\sigma} \left(\frac{v\left(D_t/C_t\right)}{v\left(D_{t-1}/C_{t-1}\right)}\right)^{1/\rho - 1/\sigma} \times \left(R_t^w\right)^{1-1/\kappa}\right]^{\kappa} \tag{26}$$

where

$$v\left(D_t/C_t\right) = \left[1 - \alpha + \alpha \left(D_t/C_t\right)^{1-1/\rho}\right]^{1/(1-1/\rho)}. \tag{27}$$

Here, β is the representative agent's subjective discount factor, $\sigma \geq 0$ is the elasticity of intertemporal substitution, $\gamma > 0$ is the coefficient of relative risk aversion, $\rho \geq 0$ is the elasticity of substitution between the durable and nondurable consumption goods and $\kappa = (1-\gamma)/(1-1/\sigma)$. The factors are the (gross) return on the wealth portfolio (R_t^w), durable goods consumption (D_t) and nondurable goods consumption (C_t).

In order to estimate and test the durable consumption model, I replicate the estimation procedure used in the estimation of the production–based model. I substitute the IMRS given in equation (26) in the moment condition for excess returns (23) and normalize the mean of the IMRS to one (including the parameter β). Finally, I fix $\alpha = 0.5$, which is the parameter estimate reported in GKY, because the nonlinear estimation of the model using only the moment conditions for excess returns revealed problems in separately identifying this parameter from ρ. GKY separately identify these parameters in practice from the intratemporal first order condition between durable and nondurable consumption and from the moment condition for the risk-free rate. I ignore these moment conditions and focus on the moment conditions for excess returns in order to make the results comparable with those from the production-based model. I note however, that the pricing errors reported here (which are the focus of the comparison) are not sensitive to reasonable changes in the value of α. The data for the pricing factors is from GKY.

[Insert Table 5 here]

Table 5 presents the first and second stage GMM estimates and tests of the Yogo (2006) durable consumption model on the same set of test assets used in the estimation of the production-based model. Across all test assets, the model is not rejected (the minimum second stage p-val of 12.5% on the 5 industry portfolios). In terms of cross-sectional R^2, the production-based model compares favorably with the durable consumption model in all the portfolio sorts except on the 9 risk sorted portfolios. The mean absolute pricing errors are higher for the durable consumption model for the 6 Fama-French portfolios (1.4% vs 1.8%) but are lower for all the other test assets.

The parameter estimates obtained here are very similar, but not identical, to those reported in GKY. Focusing on the estimation on the 5 industry portfolios, which are the test assets used in GKY, the elasticity of intertemporal substitution (IES-σ) is estimated to be slightly lower here (0.66 vs 0.46), the estimate of coefficient of relative risk aversion (RRA-γ) is estimated to be similar (16.5 vs 16.3), and the elasticity of substitution between the durable and nondurable consumption goods (EDC$-\rho$) is estimated to be slightly higher (0.97 vs 0.6). The standard errors that I obtain here are larger, and this is reflected in the variation of the parameter estimates across test assets. The difference between the parameter estimates reported here and those reported in GKY are likely the result of two differences in the estimation procedure. First, the set of moment conditions is different, as discussed above. Second, the vector of instrumental variables is also different, since I only use a constant and the dividend-yield, whereas GKY considers a larger set of instrumental variables.

It is also interesting to compare the time series of the estimated IMRS implied by the durable consumption model with the estimated marginal rate of transformation implied by the production-based model. In theory, if markets are complete, both series should be equal, since both identify the unique stochastic discount factor in the economy. Even though we cannot pin down the mean of these series from the estimation on excess returns, it is still interesting to examine the behavior of these two variables over time. This is shown in Figure 4. Interestingly, across the whole sample period, the two series have a relatively low correlation (0.24), suggesting that the information content of the two models is different. This is natural since the factors used in both models are related but they are not identical. It is also interesting to note that the low correlation is mostly driven by the different behavior of the two series in the pre 1951 period. If both model are estimated in the post 1951 sample (as in Yogo, 2006), the correlation between the two series increases to 0.52 (results not reported here but available upon request).

Taken together, the results in this section shows that the fit of the production-based model compares well with that from the Yogo (2006) durable consumption model. Both models capture reasonably well the cross-sectional variation in the returns of the test assets considered here with plausible parameter values. This result is comforting because, in theory, the performance of both models should be identical. The interpretation of the estimation results from each model is quite different, however, which makes both approaches useful in practice. The estimation of the Yogo (2006) model provides information about the preferences of the agents and links the asset pricing facts to the properties of the consumption data, while the estimation of the production-based model provides information about the properties of the firm's technology and links the asset pricing facts to the properties of production data.

6.4 Pricing Other Assets

As an out of sample diagnostic of the model, I investigate if the estimated marginal rate of transformation can price other assets not included in the original estimation of the production-based model and for a different sample period. This diagnostic represents a significant hurdle and many successful asset pricing models dramatically fail this test (see Daniel and Titman, 2005, for some examples). Here, I consider the excess returns of two different sets of portfolios that correspond to two different asset classes: (ii) 5 bond portfolios from CRSP-Fama with data available from 1952-2007 (Bonds); and (ii) 8 currency portfolios from Lustig and Verdelhan (2006) with data available from $1953-2002$ (Currency).

Appendix A-5 provides the data sources of the two sets of portfolios. The bond portfolios contain bonds with maturities between 1 and 2 years (portfolio 1), 2 and 3 years, 3 and 4 years, 4 and 5 years, and 5 and 10 years (portfolio 5). The currency portfolios are sorted on the basis of the country's interest rate level. Here, portfolio 1 contains the lowest interest rate currencies and portfolio 8 contains the highest interest rate currencies. The out of sample predicted excess returns of each portfolio are computed from $\mathbb{E}[R_t^e] = -\mathrm{Cov}(M_t, R_t^e)$, in which R_t^e is a vector with the excess returns of the set of portfolios and M_t is the estimated marginal rate of transformation.

The top left panel in Table 5 shows that the production-based model captures reasonably well the increasing monotonic relationship in the excess returns of the bond portfolios. As in the data, the predicted average excess return of the portfolio composed of high maturity bonds is higher than the average excess returns of the portfolio composed of low maturity bonds. The model slightly underestimates the average returns of these portfolios, however. The out of sample fit across currency portfolios is more modest. The model successfully captures the higher risk, and thus the higher average excess returns, of the high interest rate countries portfolio (portfolios 7 and 8), but the model fails to capture the low risk of the low interest rate country portfolios (portfolios 1,2 and 3). The pricing errors of the mid interest rate country portfolios is reasonable for portfolios 4 and 6 but large for portfolio 5.

Taken together, the diagnostic presented here suggests that the parameter estimates of the production-based model are stable over time and across asset classes if the out of sample test is based on bond portfolios, but not if it is based on currency portfolios. Thus the returns of the currency portfolios represent a puzzle for the production-based model.

6.5 Time-Varying Equity Premium and Sharpe Ratio

As a final diagnostic of the plausibility of the parameter estimates of the production-based model, I investigate if the production-based model can capture the volatile and countercyclical equity premium and Sharpe ratio of the aggregate stock market, as discussed in section 3.3. To investigate these facts, I use the estimated time series of the marginal rate of transformation and estimate a time series process for its dynamics.

In order compute the conditional equity premium and Sharpe ratio in equations (18) and (19), I impose a constant correlation between the marginal rate of transformation and stock returns, as well as constant conditional volatility of the stock market. I set these moments at their sample values, $\rho(R_t^s, M_t) = -0.44$ and $\sigma(R_t^s) = 20\%$. Although there is evidence in favor of time varying volatility in aggregate stock market returns for daily or monthly data (Bollerslev, 1986) this evidence is weaker at lower frequencies such as the annual data that I use here (Campbell, 2003). Given this assumption, all the variation in the equity premium and Sharpe ratio is thus attributed to the variation in the market price of risk, given by $\sigma_{t-1}(M_t)/\mathbb{E}_{t-1}(M_t)$. To compute this moment, I assume that the marginal rate of transformation M_t has a conditional log normal distribution with time varying conditional volatility. In this case, the conditional market price of risk is given by

$$\frac{\sigma_{t-1}(M_t)}{E_{t-1}(M_t)} = \sqrt{\exp\left(\sigma_{m,t-1}^2\right) - 1}, \tag{28}$$

in which $\sigma_{m,t-1}^2$ is the time $t-1$ conditional volatility of the log marginal rate of transformation. Given the log normal distribution assumption, time varying conditional volatility is thus important in order to generate any variation in the market price of risk over time. Using the estimated marginal rate of transformation, I estimate an AR(1) process for the mean and a GARCH(1,1) process for the conditional volatility of the demeaned log marginal rate of transformation.[14] The estimated processes are

$$\begin{aligned} m_t &= \rho_m m_{t-1} + \varepsilon_{m,t} \\ \sigma_{m,t}^2 &= \sigma + \alpha\varepsilon_{m,t-1}^2 + \beta\sigma_{m,t-1}^2, \end{aligned}$$

where $m_t = \log(M_t)$. Table 6, reports the estimation results. The coefficients $\rho_{m,}$ α and β are significant at the 10% level of significance.[15] Using these estimates, the last three rows in Table 1 provide the summary statistics for the predicted conditional equity premium, conditional market Sharpe ratio, and conditional market price of risk, which follows from equations (18), (19) and (28) respectively. Consistent with the empirical evidence, the estimated process for the marginal rate of transformation generates a market price of risk that is volatile and, using the NBER-designated business cycle recession dates, countercyclical . The conditional Sharpe ratio and the equity premium inherits the properties of the conditional market price of risk, and thus are also qualitatively consistent with the empirical evidence.

[Insert Table 6 here]

[14]Note that this process does not have any implications for the properties of the risk free rate. The marginal rate of transformation used here only measures the component of the stochastic discount factor that varies across states of nature, as discussed in Section 3.1.

[15]In the estimation of the GARCH(1,1) process for the conditional volatility, the top and bottom 5% of the first stage residuals from the AR(1) process for the mean process were winsorized. This procedure reduces the influence of the large spikes in the residuals observed in the pre second world war period, thus facilitating the analysis of the variation of the conditional volatility over time. Also note that the standard errors reported in Table 6 are not corrected for the fact that the marginal rate of transformation is an estimated variable.

Because the NBER-designated business cycle recession dates do not capture the degree of severity of each recession, Figure 6 plots the estimated time series of the conditional equity premium and of the observed dividend yield in the US economy. It is well known that the dividend yield is positively correlated with risk premia (see references in Section 3.3), and thus the dividend yield is a commonly used proxy of economic conditions (”bad times”). As such, we should see a positive relationship between the dividend yield and the estimated conditional equity premium. Figure 6 provides a visual description of the close link between the two variables. The correlation between the changes in the predicted equity premium and the observed dividend yield is 0.35. The GARCH(1,1) specification implies that large innovations in the marginal rate of transformation (e.g. 1950) translate into an higher conditional volatility of the marginal rate of transformation for several periods (and hence into an higher conditional equity premium) and the dividend yield surprisingly matches this pattern.

Naturally, the results in this section have to be interpreted with some caution since the specification of the time series process for the marginal rate of transformation is not a prediction from the model. The positive results reported here however, suggests that the estimated marginal rate of transformation has reasonable properties.

7 Conclusion

I recover a stochastic discount factor for asset returns from equilibrium marginal rates of transformation, inferred from the producer’s first order conditions. I propose a procedure for measuring the marginal rate of transformation in the data. The procedure implies a novel macro-factor asset pricing model in which the pricing factors are relative movements in industry output and price data. I test the model in the data and show that the marginal rate of transformation captures reasonably well the risk and return trade-off of several portfolio sorts, including the size and the value premia, with plausible parameter values of the firms’ production technology..

The economic interpretation of the empirical results that I obtain here suggests that the producers’ ability to transform output across states of nature is high, a result that is in contrast with what it typically assumed in standard aggregate representations of the firms’ production technology. Future research can examine the robustness of this conclusion by incorporating the smooth production function studied in this paper in a fully specified general equilibrium model and verify if we can generate artificial time series that simultaneously match the asset prices and business cycle (consumption and sectoral output, investment and good prices) facts in the data.

Finally, the production-based model developed here can be extended in several directions. One direction is to examine the pricing implications of the model for other assets, such as bonds, derivatives or the term-structure of interest rates. Another possible direction is to examine the importance of production side features such as production externalities, learning by doing, gestation lags, adjustment costs, multiple production inputs such as labor, etc., by incorporating these features into the approach developed here and investigate if these features generate measured marginal rates of transformations with better pricing properties. Understanding which production side features helps for asset pricing can also be useful for improving the specification of current general equilibrium asset pricing models.

APPENDIX

A-1 Producer's Maximization Problem

Define the vector of state variables as $x_{it-1} = (K_{it-1}, \epsilon_{it-1}, P_{it-1}, Z_{it-1})$ where K_{it-1} is the current period stock of capital, ϵ_{it-1} is the current period productivity level and $P_{it-1} = p_{it-1}/p_{t-1}$ is the current period relative price of good i with respect to the price of a numeraire good (p_{t-1}). The variable Z_{it-1} summarizes the information about the next period distribution (i.e. state-by-state values and probabilities) of the stochastic discount factor M_t, the underlying productivity level Θ_{it} and the relative price of good i, P_{it}. Let $V(x_{it-1})$ be the present value of firm i at the end of period $t-1$ given the vector of state variables x_{it-1}. The Bellman equation of the producer is

$$V(x_{it-1}) = \max_{\{I_{it-1}, \epsilon_{it}\}} \{D_{it-1} + \mathbb{E}_{t-1}[M_t V(x_{it})]\}$$

subject to the constraints,

$$\begin{aligned} D_{it-1} &= P_{it-1} Y_{it-1} - I_{it-1} \\ Y_{it-1} &= \epsilon_{it-1} F^i(K_{it-1}) \\ 1 &\geq \mathbb{E}_{t-1}\left[\left(\frac{\epsilon_{it}}{\Theta_{it}}\right)^{\alpha}\right]^{\frac{1}{\alpha}} \qquad (1) \\ K_{it} &= (1-\delta_i)K_{it-1} + I_{it-1} \end{aligned}$$

for all dates t. $\mathbb{E}_{t-1}[.]$ is the expectation operator conditional on the firm's information set at the end of period $t-1$, δ_i is the depreciation rate of capital and $F^i(.)$ is the (certain) production function, which is increasing and concave.

Substitute the law of motion for capital in the value function and let μ_{it-1} be the Lagrange multiplier associated with the technological constraint in equation (1), the first order conditions are

$$\frac{\partial}{\partial I_{it-1}} : \mathbb{E}_{t-1}[M_t V_k(x_{it})] = 1 \qquad (2)$$

$$\frac{\partial}{\partial \epsilon_{it}} : M_t V_{\epsilon_i}(x_{it}) = \mu_{it-1}\mathbb{E}_{t-1}\left[\left(\frac{\epsilon_{it}}{\Theta_{it}}\right)^{\alpha}\right]^{\frac{1}{\alpha}-1} \epsilon_{it}^{\alpha-1}\Theta_{it}^{-\alpha}$$

Since in equilibrium the restriction in equation (1) is naturally binding, we have $\mathbb{E}_{t-1}\left[\left(\frac{\epsilon_{it}}{\Theta_{it}}\right)^{\alpha}\right] = 1$. Substituting this in the previous equation, we can write the first order condition for the optimal choice of the productivity level ϵ_{it} as

$$\frac{\partial}{\partial \epsilon_{it}} : M_t V_{\epsilon_i}(x_{it}) = \mu_{it-1} \epsilon_{it}^{\alpha-1} \Theta_{it}^{-\alpha} \tag{3}$$

The envelope conditions are

$$V_{k_i}(x_{it-1}) = P_{it-1}\epsilon_{it-1} F^i_{k_i}(K_{it-1}) + \mathbb{E}_{t-1}[M_t V_{k_i}(x_{it})](1-\delta_i) \tag{4}$$

$$V_{\epsilon_i}(x_{it-1}) = P_{it-1} F^i(K_{it-1}) \tag{5}$$

Using equation (2), the envelope condition (4) can be written as

$$V_{k_i}(x_{it-1}) = P_{it-1}\epsilon_{it-1} F^i_k(K_{it-1}) + (1-\delta_i) \tag{6}$$

Substituting the envelope condition (5) at time t back into equation (3) yields

$$M_t P_{it} F^i(K_{it}) = \mu_{it-1} \epsilon_{it}^{\alpha-1} \Theta_{it}^{-\alpha} \tag{7}$$

Taking expectations on both sides of the previous equation yields

$$\mathbb{E}_{t-1}\left[M_t P_{it}\right] F^i(K_{it}) = \mu_{it-1} \mathbb{E}_{t-1}\left[\epsilon_{it}^{\alpha-1} \Theta_{it}^{-\alpha}\right] \tag{8}$$

This equation defines the Lagrange multiplier. Substitute μ_{it-1} from equation (8) back in equation (7) yields

$$M_t P_{it} F^i(K_{it}) = \mathbb{E}_{t-1}\left[M_t P_{it}\right] F^i(K_{it}) / \mathbb{E}_{t-1}\left[\epsilon_{it}^{\alpha-1} \Theta_{it}^{-\alpha}\right] \epsilon_{it}^{\alpha-1} \Theta_{it}^{-\alpha} \tag{9}$$

Rearranging terms

$$M_t = \mathbb{E}_{t-1}\left[M_t P_{it}/P_{it-1}\right] / \mathbb{E}_{t-1}\left[(\epsilon_{it}/\epsilon_{it-1})^{\alpha-1} (\Theta_{it}/\Theta_{it-1})^{-\alpha}\right] \left(\frac{P_{it-1}}{P_{it}}\right) \left(\frac{\epsilon_{it}}{\epsilon_{it-1}}\right)^{\alpha-1} \left(\frac{\Theta_{it}}{\Theta_{it-1}}\right)^{-\alpha}$$

which we can write compactly as

$$M_t = \phi_{it-1} \left(\frac{P_{it-1}}{P_{it}}\right) \left(\frac{\epsilon_{it}}{\epsilon_{it-1}}\right)^{\alpha-1} \left(\frac{\Theta_{it}}{\Theta_{it-1}}\right)^{-\alpha}$$

where $\phi_{it-1} = \mathbb{E}_{t-1}\left[M_t P_{it}/P_{it-1}\right] / \mathbb{E}_{t-1}\left[(\epsilon_{it}/\epsilon_{it-1})^{\alpha-1} (\Theta_{it}/\Theta_{it-1})^{-\alpha}\right]$. This is equation (9) in the text.

Solving for the (growth rate) in the productivity level yields

$$\frac{\epsilon_{it}}{\epsilon_{it-1}} = \phi_{it-1}^{\frac{1}{1-\alpha}} \left(\frac{\Theta_{it}}{\Theta_{it-1}}\right)^{\frac{\alpha}{\alpha-1}} \left(\frac{M_t P_{it}}{P_{it-1}}\right)^{\frac{1}{\alpha-1}},$$

which is equation (6) in the text.

Finally, to obtain the expression for investment returns, substitute equation (6) at time t back in equation (2) to obtain

$$\mathbb{E}_{t-1}[M_t R_t^I] = 1$$

where

$$R_t^I = (1-\delta_i) + P_{it}\epsilon_{it}F_k^i(K_{it})$$

is the (random) investment return. These are equations (10) and (11) in the text.

The second order conditions are satisfied by the assumptions on the production technology, i.e., $\alpha > 1$ and $F^i(.)$ increasing and concave.

A-2 Proof of Proposition 1

From the producer i first order conditions (see equation (9) in the text) we have

$$M_t = \bar{\phi}_{it-1}\left(\frac{P_{it-1}}{P_{it}}\right)\left(\frac{Y_{it}}{Y_{it-1}}\right)^{\alpha-1}\left(\frac{\Theta_{it}}{\Theta_{it-1}}\right)^{-\alpha}, \tag{10}$$

where $\bar{\phi}_{it-1} = \mathbb{E}_{t-1}\left[M_t P_{it}/P_{it-1}\right]/\mathbb{E}_{t-1}\left[(Y_{it}/Y_{it-1})^{\alpha-1}(\Theta_{it}/\Theta_{it-1})^{-\alpha}\right]$. Since markets are complete, the SDF M_t is unique. This implies that at an interior solution, the marginal rate of transformation is equalized across time and states across all technologies $i = 1,..,N$. Taking the log of both sides of the previous equation we have

$$m_t = \gamma_{it-1} - \Delta\bar{p}_{it} + (\alpha-1)\Delta y_{it} - \alpha\Delta\theta_{it} \text{ for } i = 1,..,N \tag{11}$$

where lowercase variables are the log of the corresponding uppercase variable, $\gamma_{it-1} = \ln\left(\bar{\phi}_{it-1}\right)$, $\theta_{it} = \ln(\Theta_{it})$ and Δ is the first difference operator. I use a bar over the log relative price ($\bar{p}_i$) to emphasize that this is a *relative* price of firm's i output (with respect to the numeraire good), not the actual price. According to the identification Assumption 1 we have

$$\alpha\Delta\theta_{it} = \sum_{j=1}^{J}\lambda_{ij}F_t^j \tag{12}$$

where J is the number of common productivity factors in the economy. Substituting equation (12) in equation (11) yields

$$m_t = \gamma_{it-1} - \Delta\bar{p}_{it} + (\alpha-1)\Delta y_{it} - \sum_{j=1}^{J}\lambda_{ij}F_t^j \text{ for } i = 1,..,N \tag{13}$$

As specified in Assumption 1, I normalize $\lambda_{1j} = 1$ for $j = 1,..,J$. Now, consider the first order conditions for $J+1$ technologies, with $J+1 \leq N$ (i.e. the number of technologies is stricly larger than the number of common productivity factors). Taking the difference between equation (13) for technologies $i = 2, ..., J + 1$ relative to the same equation for technology 1 yields

$$0 = \left[\gamma_{it-1} - \gamma_{1t-1}\right] - \left[\Delta p_{it} - \Delta p_{1t}\right] + (\alpha - 1)\left[\Delta y_{it} - \Delta y_{1t}\right] - \sum_{j=1}^{J} (\lambda_{ij} - 1) F_t^j \quad \text{for } i = 2, .., J+1 \tag{14}$$

where I've used the notation p_{it} (without the bar on the top) to denote the actual price (not relative). The fact that I now use the actual price follows from the fact that the price of the numeraire good cancels out from the difference in the relative prices between any two goods.

From now on, it is convenient to write all the $i = 2, .., J + 1$ equations defined in (14) in matrix form. Rearranging terms we have

$$LF_t = \Omega_{t-1} - I\Delta P_t + (\alpha - 1)I\Delta Y_t \tag{15}$$

where I is a $[J \times J]$ identity matrix, Ω_{t-1} is a dimension J column vector in which each element i is $\gamma_{(i+1)t-1} - \gamma_{1t-1}$, L is a $[J \times J]$ matrix in which each row-i, column-j element is given by $\lambda_{(i+1)j} - 1$, ΔP_t is a dimension J column vector in which each element i is $\Delta p_{(i+1)t} - \Delta p_{1t}$, ΔY_t is a dimension J column vector in which each element i is $\Delta y_{(i+1)t} - \Delta y_{1t}$ and F_t is a dimension J column vector in which each element i is F_t^i.

With $J \geq 1$ common productivity factors, it is easy to show that we can identify these common factors from price and output data from $N = J + 1$ technologies. Assuming the matrix L has full rank, we can solve equation (15) for the size J vector F_t to obtain

$$F_t = L^{-1}\Omega_{t-1} - L^{-1}\Delta P_t + (\alpha - 1)L^{-1}\Delta Y_t \tag{16}$$

This equation shows that we can identify the underlying common productivity factors from price and output data only. As an example, for the case of one common productivity factor ($J = 1$) we have $L = \lambda_{21} - 1$, and thus the single common productivity factor F_t can be recovered from

$$F_t = (\lambda_{21} - 1)^{-1}\left[\gamma_{2t-1} - \gamma_{1t-1}\right] - (\lambda_{21} - 1)^{-1}\left[\Delta p_{2t} - \Delta p_{1t} - (\alpha - 1)(\Delta y_{2t} - \Delta y_{1t})\right] \tag{17}$$

To express the actual marginal rate of transformation in terms of observed price and output data, substitute equation (16) in the marginal rate of transformation defined in equation (13) for technology 1 to obtain

$$m_t = \gamma_{1t-1} - \Delta \bar{p}_{1t} + (\alpha - 1)\Delta y_{1t} - \iota_J \left\{L^{-1}\Omega_{t-1} - L^{-1}\Delta P_t + (\alpha - 1)L^{-1}\Delta Y_t\right\}$$

where ι_J is a size J row vector of ones. Finally, the previous equation can be written more compactly as

$$m_t = \varkappa_{t-1} - \sum_{i=2}^{J+1} \left[b_i^p \left(\Delta p_{it} - \Delta p_{1t} \right) + b_i^y \left(\Delta y_{it} - \Delta y_{1t} \right) \right] - \Delta \bar{p}_{1t} + (\alpha - 1) \Delta y_{1t} \tag{18}$$

where b_i^p and b_i^y are the the $(i-1)^{th}$ elements in the $[1 \times J]$ row vectors $-\iota_J L^{-1}$ and $(\alpha-1)\iota_J L^{-1}$ respectively, and $\varkappa_{t-1} = \gamma_{1t-1} - \iota_J L^{-1} \Omega_{t-1}$ is a variable pre-determined at t. Equation (18) is the exact log marginal rate of transformation. For empirical purposes, this marginal rate of transformation can be further simplified. As I show empirically, the factor risk prices b^p's are typically a large number and the parameter α is small (close to one). Thus, to an excellent approximation, the previous marginal rate of transformation can be written as (and taking the exponential)

$$M_t \approx \kappa_{t-1} \exp \left[- \sum_{i=2}^{J+1} \left[b_i^p \left(\Delta p_{it} - \Delta p_{1t} \right) + b_i^y \left(\Delta y_{it} - \Delta y_{1t} \right) \right] \right] \tag{19}$$

where $\kappa_{t-1} = \exp(\varkappa_{t-1})$. This equation shows that in order to empirically identify the marginal rate of transformation, only the *relative* movements (with respect to the reference technology 1) in output and price growth matter. For the case of one common productivity factor ($J = 1$), we have $L^{-1} = (\lambda - 1)^{-1}$ and thus we can write the marginal rate of transformation as

$$\mathrm{M_t} \approx \kappa_{t-1} \exp \left[-b^p \left(\Delta p_{NDt} - \Delta p_{Dt} \right) - b^y \left(\Delta y_{NDt} - \Delta y_{Dt} \right) \right]$$

where, to simplify notation, I've defined $\lambda_{21} = \lambda$, and the factor risk prices are given by

$$\begin{bmatrix} b^p \\ b^y \end{bmatrix} = \begin{bmatrix} 1/(1-\lambda) \\ (\alpha - 1)/(\lambda - 1) \end{bmatrix}$$

This completes the proof.

A-3 Principal Components Analysis

To do a principal components analysis of the cross section of the relative price and output growth, I first linearize the nonlinear marginal rate of transformation in Proposition 1 by a first order Taylor expansion around the unconditional mean of the factors, which I denote by $\mathbb{E}[x_t]$ where x_t is the factor. Then, normalizing the mean of the marginal rate of transformation to one (since the mean is not identified from the estimation of the model on excess returns) yields

$$M_t \approx 1 - \sum_{i=2}^{J+1} \left[b_i^p \left(\Delta p_{it} - \Delta p_{1t} - \mathbb{E}\left[\Delta p_{it} - \Delta p_{1t} \right] \right) + b_i^y \left(\Delta y_{it} - \Delta y_{1t} - \mathbb{E}\left[\Delta y_{it} - \Delta y_{1t} \right] \right) \right] \quad (20)$$

I then do a separate principal components analysis of the cross section of relative price growth $(\Delta p_{it} - \Delta p_{1t})_{i=2}^{J+1}$ and of the cross section of relative output growth $(\Delta y_{it} - \Delta y_{1t})_{i=2}^{J+1}$ (see Mardia, Kent and Bibby, 1979, for a textbook treatment of principal components analysis. By construction, the first principal component is the orthogonal component that explains most of the variation in the output or price growth in all sectors, the second component explains most of the part not explained by the first component and so forth. Once the principal components have been extracted, each pricing factor in equation (20) can specified as a linear combination of the principal components as

$$\Delta p_{it} - \Delta p_{1t} = \sum_{j=1}^{J} \gamma_{ij}^p PPC_j \ , i = 2, .., J+1 \quad (21)$$

$$\Delta y_{it} - \Delta y_{1t} = \sum_{j=1}^{J} \gamma_{ij}^y OPC_j \ , i = 2, .., J+1 \quad (22)$$

where PPC_j is the jth principal component of the cross section of relative prices growth, OPC_j is the jth principal component of the cross section of relative output growth and γ_{ij}^P and γ_{ij}^y are the loadings of each pricing factor on the corresponding principal component.

In the empirical section, I consider all the four sectors reported in NIPA, namely durable goods, nondurable goods, services and structures. As in the one common productivity factor specification, I specify the durable goods sector as the technology 1 (the reference technology). The first principal component of the relative output growth factor alone explains 73% of the total variance. The first principal component of the cross section of relative price growth explains almost 66% of the total variance in the cross section of relative price growth (detailed results available upon request). These results suggests that the marginal rate of transformation is well approximated by the first principal components of the output growth and of the relative price growth. Thus, under the multiple common productivity factors specification, the marginal rate of transformation is approximately given by

$$M_t \approx 1 - b^p \text{PFPC}_t - b^y \text{OFPC}_t \quad (23)$$

as reported in the text, where PFPC_t (price first principal component) is the first principal component of the cross-section of the relative price growth and OFPC_t (output first principal component) is the first principal component of the cross-section of relative output growth. This approximation is obtained by first noting that using only the first principal component, the relative output and relative price factors in equations (21) and (22) are approximately given by $\Delta p_{it} - \Delta p_{1t} \approx \gamma^p_{i1} PFPC_t$ and $\Delta y_{it} - \Delta y_{1t} = \gamma^y_{i1} OFPC_t$. Substituting these expressions in equation (20) yields equation (23), where the factor risk prices are given by

$$\begin{bmatrix} b^p \\ b^y \end{bmatrix} = \begin{bmatrix} \sum_{i=2}^{N} b^p_i \gamma^p_{i1} \\ \sum_{i=2}^{N} b^y_i \gamma^y_{i1} \end{bmatrix}$$

A-4 Estimation Methodology

Estimation is by the Generalized Method of Moments (GMM), following the methodology developed by Hansen and Singleton (1982) Let z_{t-1} be a $I \times 1$ vector of instrumental variables known at time $t-1$ and R^e_{it} $(i = 1, .., N)$ be excess returns on N portfolios. I normalize the mean of the stochastic discount factor to one ($\mathbb{E}[M_t] = 1$). The following moment condition is used for estimation and testing,

$$0 = \mathbb{E}[M_t R^e_{it} z_{t-1}] \ , \ (i = 1, .., N). \tag{24}$$

Equation (24) represents NI moment conditions. In the two empirical specifications of the production-based model, there are $k = 2$ parameters (α and λ) in the one common productivity factor specification, and $k = 3$ parameters (the factor risk prices b^p and b^y and the constant c^{te}) in the multiple common productivity factor specification. The $N - k$ overidentifying restrictions of the model are tested by the $J-$test (Hansen, 1982). This test is conceptually similar to the standard GRS test of alphas (Gibbons, Ross and Shanken, 1989), applied to GMM cross-sectional regressions.

The instruments z_{t-1} are a constant and the dividend-price ratio. I normalize the mean of the dividend-price ratio to one. I report both first stage and second stage (efficient) GMM estimates. In the first stage I use the identity matrix as the weighting matrix, while in the second stage I use the inverse of the Newey-West estimate of the covariance matrix of the sample pricing errors in the first stage. In the estimation of this matrix, I use three period lags to account for the possibility of time aggregation in output and price data (see discussion in Hall , 1988, on consumption data).

As a diagnostic of the fit of the estimated model, I report two commonly used, albeit imperfect, measures: the cross-sectional R-squared (R^2) and the mean absolute pricing error

(MAE).[16] The R^2 is obtained from a an OLS regression of the realized excess returns on the predicted excess returns by the model including a constant. The MAE is obtained by first computing the pricing error of each asset i, $\alpha_i = \mathbb{E}[R_i^e]^{\text{observed}} - \mathbb{E}[R_i^e]^{\text{Predicted}}$, and then take the average across assets of the absolute value of the pricing errors to obtain MAE= $\frac{1}{N}\sum_{i=1}^{N}\text{Abs}(\alpha_i)$, where N is the number of test assets and Abs(α_i) is the absolute value of the pricing error of asset i. In computing these measures, I use the parameter estimates of the model in which only the constant is used as instrument.

Following Gomes, Kogan and Yogo (2009), in matching returns with macro variables such as output, price or consumption data, I use the Campbell's (2003) "beginning of period" timing convention. Specifically, the asset return at time t is matched with the growth rate of the macro variable at time $t + 1$. A convention is needed because the level of the macro variable is a flow during a year rather than a point-in-time observation as the returns; that is, macro data are time averaged. The "beginning of period" timing convention assumes that output data for year t (and for the other macro variables) measures the output at the beginning of the year. In this case, the relevant growth rate of output for a given year is next year's output divided by this year's output. This matching convention improves the performance of all the models estimated here relative to the "end of period" timing convention.[17]

A-5 Description of the Asset Data

The data for the dividend-yield is from Robert Shiller's webpage. The data for the 6 Fama-French portfolios sorted on size and book to market is from Kenneth French's webpage. The data used to compute the 9 risk pre-ranking beta double sorted portfolios is from CRSP, available at the Wharton Research Data Services (WRDS) website. The data for the 5 industry portfolios is from Gomes, Kogan and Yogo (2009). Excess returns are computed by subtracting the risk free rate, as measured by the US treasury bill return rate, from CRSP. The description of each set of portfolio sorts is the following:

6 Fama-French portfolios sorted on size and book-to-market: these portfolios are constructed at the end of each June, and are the intersections of 2 portfolios formed on size (market equity, ME) and 3 portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoint for year t is the median of the NYSE market equity at the

[16] See Kandel and Stambaugh (1995), Roll and Ross (1995), and Lewellen, Nagel and Shanken (2006) for a discussion of the limitations in the R^2 measure.

[17] Jagannathan and Wang (2007) show that even though the standard consumption-based model does not perform well with annual averages, it performs significantly better when annual consumption growth is measured based only on the fourth quarter of each year. Jagannathan and Wang's paper highlights the effect of different matching assumption between returns and macroeconomic variables on asset pricing tests.

end of June of t. BE/ME for June of year t is the book equity for the last fiscal year end in t-1 divided by ME for December of $t-1$. The BE/ME breakpoints at the 30th and 70th percentiles of the cross sectional distribution of BE/ME NYSE firms. For each portfolio, the value weighted returns from July of t to June of t+1 are then computed.

9 Risk pre-ranking beta double sorted portfolios: the relative output growth and the relative price growth factors that I use here have an annual frequency and thus directly using these factors to create "pre-ranking" betas is not appropriate due to the small sample size. To address this issue, I create two mimicking portfolios (at monthly frequency) of these factors which I label price mimicking portfolio (PMP_t) and output mimicking portfolio (OMP_t). The PMP_t is obtained by first estimating the following regression,

$$\Delta p_{\mathrm{ND}t} - \Delta p_{\mathrm{D}t} = a + b' R_t^e + \varepsilon_t \tag{25}$$

where$\Delta p_{\mathrm{ND}t} - \Delta p_{\mathrm{D}t}$ is the relative price growth factor and R_t^e are the excess returns on the base assets. The coefficients b can be interpreted as the weights in a zero-cost portfolio. The return on the PMP_t is then

$$\mathrm{PMP}_t = b' R_t^e \tag{26}$$

which is the minimum variance combination of assets that is maximally correlated with the relative price growth factor. Regression (25) (and similarly for the relative output growth factor) is estimated using annual data from 1930 to 2007. Then, assuming that the coefficients b are relatively stable over time and within the year, I use equation (26) to extend the sample before 1930 and to generate observations of the mimicking portfolio at a monthly frequency. The base test assets I employ are the Fama-French 6 benchmark portfolios and the 10 momentum portfolios (I use momentum portfolios to capture components of these factors that are orthogonal to the size and book-to-market factors so that these portfolios do not necessarily span the same space of the size and book-to-market portfolios). An identical procedure is used to obtain the OMP_t factor. The correlation between each factor and the corresponding estimated mimicking portfolio is 0.32 for the price factor and 0.52 for the output factor.

Following Fama and French (1992) I then create nine pre-ranking beta double sorted risk based portfolios of NYSE, AMEX and NASDAQ stocks as follows. For every calendar year, I first estimate the PMP and the OMP betas for each firm, using 24 to 60 months of past return data. As in Fama and French (1992), I denote this beta as the "pre-ranking" PMP and OMP beta estimate. I then do the following double sorting procedure: I sort stocks into three bins (cutoffs at the 33th and 66th percentile) based on their "pre-ranking" PMP beta and repeat the procedure based on each stock's "pre-ranking" OMP beta. The intersection of these bins gives 9 portfolios. I then compute the return on each of these portfolios for the next 12 calendar months by a value weighted average of the returns of the stocks in the portfolio. This procedure is repeated at the end of June for each calendar year.

5 Industry portfolios: these portfolios were constructed by Gomes, Kogan and Yogo (2009). The universe of stocks to construct these portfolios is the ordinary common equity traded in NYSE, AMEX, or Nasdaq, which are recorded in the Center for Research in Securities Prices (CRSP) Monthly Stock Database. In June of each year t, the universe of stocks is sorted into five industry portfolios based on their SIC code: services, nondurable goods, durable goods, investment, and other industries. Other industries include the wholesale, retail, and financial sectors as well as industries whose primary output is to government expenditures or net exports. The stock must have a non-missing SIC code in order to be included in a portfolio. Once the portfolios are formed, their value-weighted returns are tracked from July of year t through June of year t + 1.

5 Bond portfolios: the five bond portfolios used in the out of sample tests are obtained from WRDS (Wharton Research Database System) at http://wrds.wharton.upenn.edu. Each bond portfolios contain bonds with maturities between 1 and 2 years (portfolio 1), 2 and 3 years (portfolio 2), 3 and 4 years (portfolio 4), 4 and 5 years (portfolio , and 5 and 10 years (portfolio 5)

8 Currency portfolios: the eight currency portfolios used in the out of sample tests are obtained from Adrien Verdelhan's webpage. The currency portfolios are sorted on the basis of the country level interest rate. Portfolio 1 contains the lowest interest rate currencies and portfolio 8 contains the highest interest rate currencies.

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Table 1
Summary Statistics

This table reports the summary statistics of the growth rate of output (ΔY_i), the growth rate of prices (ΔP_i) in sector i =D (durable goods) and sector i =ND (nondurable goods), and the growth rate in consumption (Δi) of i =D (durable goods) and i =C (nondurable goods and services). It also reports the summary statistics of the first principal components of the cross section of relative price growth (PFPC), the first principal component of relative output growth (OFPC), the estimated demeaned log marginal rate of transformation (MRT) of the one common productivity factor specification of the production-based model, the estimated common productivity factor F_t and the estimated demeanded log intertemporal marginal rate of substitution (IMRS) of the Yogo (2006) durable consumption model implied by the first stage estimation of each model on the 6 Fama-French portfolios sorted on size and book to market. Finally, the table reports the estimated conditional market price of risk, the estimated market Sharpe ratio and the estimated conditional equity premium implied by the one common productivity factor specification of the production-based model and a AR(1)-GARCH(1,1) specification of the demeaned log marginal rate of transformation. All values are in percentage except the first order autocorrelation AC(1), the MRT, IMRS and F_t. The data are annual and the sample is 1930 − 2007.

	Full Sample			NBER expansions		NBER recessions	
Variables	Mean	S.D.	AC(1)	Mean	S.D.	Mean	S.D.
ΔY_{D}	6.15	12.90	0.25	10.95	11.02	−3.83	10.73
ΔY_{ND}	2.87	2.94	0.09	3.52	2.53	1.51	3.31
ΔP_{D}	1.35	3.75	0.64	1.20	3.01	1.65	5.03
ΔP_{ND}	2.88	4.55	0.63	3.37	3.65	1.84	5.98
ΔD	2.57	3.91	0.16	3.39	3.97	0.87	3.22
ΔC	1.95	2.10	0.37	2.67	1.52	0.44	2.37
Estimated Principal Components							
PFPC-Price	0	4.32	0.60	0.89	3.59	−1.74	5.17
OFPC-Output	0	20.46	0.13	−4.64	21.51	8.83	14.66
Estimated Stochastic Discount Factors (SDF) and Common Productivity Factor							
MRT (SDF−Production)	0	0.94	0.58	−0.24	0.79	0.49	1.06
Common Factor (F_t)	0	0.89	0.58	0.23	0.75	−0.47	1.00
IMRS (SDF−Consumption)	0	0.68	0.09	−0.07	0.54	0.15	0.90
Estimated Asset Pricing Moments Implied by the Production-Based Model							
Price of Risk	0.56	0.20	0.88	0.54	0.21	0.60	0.17
Sharpe Ratio	0.24	0.09	0.88	0.24	0.09	0.26	0.07
Equity Premium	4.85	1.72	0.88	4.68	1.82	5.23	1.44

Table 2
Characteristics of 9 Risk Sorted Portfolios

This table reports the average annual value weighted excess returns (Return, in %), the post-ranking covariances (units $\times 10^{-3}$) with the relative (nondurable minus durable goods) output growth factor (Output-Cov) and the relative (nondurable minus durable goods) price growth factor (Price-Cov) with the 9 Risk double sorted on pre-ranking betas portfolios. Portfolio "High" is a portfolio of stocks whose pre-ranking beta of the corresponding factor (Output or Price) is in the top 33th percentile and portfolio "Low" is a portfolio with stocks whose pre-ranking beta of the corresponding factor is in the bottom 33th percentile. The portfolios are rebalanced annually. The data are annual and the sample is $1930 - 2007$.

	Output-Low			Output-Med			Output-High		
Price Sort	Low	Medium	High	Low	Medium	High	Low	Medium	High
Return	8.92	10.63	13.44	8.07	9.23	10.12	6.62	7.71	8.45
Output-Cov	−9.61	−11.29	−10.73	−9.90	−8.88	−9.35	−5.53	−5.60	−7.86
Price-Cov	0.85	1.58	2.16	0.97	0.95	1.55	0.58	0.81	1.58

Table 3

GMM Estimation of the Production-Based Model on Several Portfolio Sorts

This table reports the first stage (columns 1st) and second stage (columns 2nd) GMM estimates with the corresponding standard errors in parenthesis and tests of the production-based model. The estimated moment condition is $0 = \mathbb{E}[M_t R^e_{it} z_{t-1}]$ in which z_{t-1} is a vector of instrumental variables that includes a constant and the dividend-yield on the aggregate stock market, and R^e_t is a vector with the excess returns of the following portfolio sorts: (i) the 6 Fama-French portfolios sorted on size and book to market (6-Size-BM); (ii) 9 risk pre-ranking beta double sorted portfolios (9-Risk); (iii) 5 Gomes, Kogan and Yogo (2009) industry portfolios (5-Ind); and (iv) all portfolios together (20-All). Panel A reports the results for the one common productivity factor specification in which $M_t = \exp\left[-b^p\left(\Delta P_{NDt} - \Delta P_{Dt}\right) - b^y\left(\Delta Y_{NDt} - \Delta Y_{Dt}\right)\right]$ and the factor risk prices are $b^p = 1/(\lambda - 1)$ and $b^y = (\alpha - 1)/(1 - \lambda)$. Panel B reports results for the multiple common productivity factors specification in which $M_t = 1 - b^p \text{PFPC}_t - b^y \text{OFPC}_t$. This specification includes a constant (c^{te}) in the pricing equation and the factor risk prices b^p, b^y are free parameters. The table also reports the following measures of the goodness of fit and tests of the model (Diagnostics): the implied cross-sectional R-squared (R^2), the implied mean absolute pricing errors (MAE, in %) and the first and second stage J-test of overidentifying restrictions with the corresponding p-value (in %). The data are annual and the sample is $1930 - 2007$.

Panel A: One Common Productivity Factor Specification (Nonlinear Model)

	6-Size-BM		9-Risk		5-Ind		20-All	
Parameters	1st	2nd	1st	2nd	1st	2nd	1st	2nd
α	1.04	1.02	1.05	1.00	1.16	1.12	1.05	1.04
	[0.26]	[0.07]	[0.19]	[0.08]	[0.24]	[0.16]	[0.15]	[0.04]
λ	0.96	0.97	0.96	0.97	0.94	0.95	0.96	0.97
	[0.04]	[0.01]	[0.03]	[0.01]	[0.05]	[0.03]	[0.02]	[0.01]
Diagnostics								
R^2	83.9		72.6		48.0		79.0	
MAE	1.4		1.2		1.3		1.4	
J-Test	6.8	6.5	9.3	8.6	10.2	9.0	15.1	13.8
p-value (J)	74.6	77.6	90.2	93.1	25.0	34.0	99.9	99.9

Panel B: Multiple Common Productivity Factors Specification (Linear Model)

Risk Prices	1st	2nd	1st	2nd	1st	2nd	1st	2nd
c^{te}	0.02	0.02	0.04	0.03	0.03	0.07	0.03	0.03
	[0.06]	[0.03]	[0.04]	[0.02]	[0.05]	[0.03]	[0.05]	[0.01]
b^{y}	−7.42	−5.38	−5.56	−6.12	−5.69	−4.42	−6.09	−6.17
	[5.40]	[1.82]	[4.17]	[1.04]	[2.81]	[2.17]	[4.05]	[0.23]
b^{p}	13.95	19.54	8.47	3.53	8.59	−9.47	14.50	14.79
	[13.41]	[6.18]	[23.41]	[5.78]	[20.45]	[6.17]	[14.12]	[1.62]
Diagnostics								
R^2	84.1		80.6		74.9		82.8	
MAE	1.4		1.0		0.9		1.1	
J-Test	11.0	11.2	11.7	11.4	5.9	6.4	18.3	18.3
p-value (J)	27.5	26.5	70.1	72.3	54.8	49.1	99.6	99.6

Table 4
Characteristics of 6 Size and Book-to-Market Portfolios

This table reports the average annual value weighted excess returns (Return, in %), the post-ranking covariances (units $\times 10^{-3}$) with the relative (nondurable minus durable goods) output growth factor (Output-Cov) and the relative (nondurable minus durable goods) price growth factor (Price-Cov) with the Fama-French 6 Size and Book-to-Market (BM) portfolios. The portfolios are rebalanced annually. The data are annual and the sample is $1930 - 2007$.

	BM-Growth		BM-Neutral		BM-Value	
Size Sort	Small	Big	Small	Big	Small	Big
Return	10.24	7.26	14.01	8.88	16.62	12.07
Output-Cov	−10.09	−7.82	−10.61	−8.94	−10.76	−11.03
Price-Cov	1.34	1.00	1.78	1.50	2.48	1.88

Table 5
GMM Estimation of the Yogo (2006) Durable Consumption Model

This table reports the first stage (columns 1^{st}) and second stage (columns 2^{nd}) GMM estimates with the corresponding standard errors in parenthesis and tests of the Yogo (2006) durable consumption model. The estimated moment condition is $0 = \mathbb{E}[M_t R^e_{it} z_{t-1}]$ in which z_{t-1} is a vector of instrumental variables that includes a constant and the dividend-yield on the aggregate stock market, and R^e_t is a vector with the excess returns of the following portfolio sorts: (i) the 6 Fama-French portfolios sorted on size and book to market (6-Size-BM); (ii) 9 risk pre-ranking beta double sorted portfolios (9-Risk); (iii) 5 Gomes, Kogan and Yogo (2009) industry portfolios (5-Ind); and (iv) all portfolios together (20-All). The stochastic discount factor M_t is the intertemporal marginal rate of substitution given in equation (26). The estimated parameters are the elasticity of intertemporal substitution (EIS-σ), the coefficient of relative risk aversion (RRA-γ) and the elasticity of substitution between the durable and nondurable consumption goods (EDC-ρ). The table reports the following measures of the goodness of fit and tests of the model (Diagnostics): the implied cross-sectional R-squared (R^2), the implied mean absolute pricing errors (MAE, in %) and the first and second stage J-test of overidentifying restrictions with the corresponding p-value (in %). The data are annual and the sample is $1930 - 2007$.

	6-Size-BM		9-Risk		5-Ind		20-All	
Parameters	1^{st}	2^{nd}	1^{st}	2^{nd}	1^{st}	2^{nd}	1^{st}	2^{nd}
EIS-σ	0.44	0.33	0.12	0.14	0.46	0.41	0.28	0.24
	[0.67]	[0.23]	[0.24]	[0.11]	[0.37]	[0.24]	[0.23]	[0.06]
RRA-γ	13.67	20.46	20.56	27.38	16.54	22.29	18.89	24.49
	[19.44]	[10.46]	[9.59]	[8.81]	[15.42]	[13.74]	[10.63]	[5.82]
EDC-ρ	0.11	0.08	0.08	0.29	0.97	1.05	0.39	0.39
	[0.26]	[0.18]	[7.10]	[0.17]	[1.07]	[0.70]	[0.43]	[0.12]
Diagnostics								
R^2	65.5		79.5		5.0		53.5	
MAE	1.8		0.7		1.1		1.3	
J-Test	9.5	11.166	9.4	10.2	11.2	11.323	17.1	17.1
p-value (J)	39.1	26.451	85.4	80.1	12.7	12.514	99.7	99.7

Table 6
Estimation of the Marginal Rate of Transformation Dynamics

This table reports the estimates and corresponding standard errors in parenthesis of an AR(1)-GARCH(1,1) process for the demeaned log marginal rate of transformation implied by the first stage GMM estimation of the one common productivity factor specification of the production-based model on the 6 Fama-French portfolios sorted on size and book to market. The process for the mean of the (demeaned) log marginal rate of transformation is $m_t = \rho_m m_{t-1} + \varepsilon_{m,t}$ and the process for the conditional volatility is $\sigma^2_{m,t} = \sigma + \alpha\varepsilon^2_{m,t-1} + \beta\sigma^2_{m,t-1}$. The data are annual and the sample is $1930 - 2007$.

	ρ_m	σ	α	β
Estimates	0.55	0.02	0.23	0.72
	[6.14]	[0.99]	[1.94]	[4.93]

Figure 1
Production-Based Approach and Production Possibilities Frontier

This figure shows the plot of the production possibilities frontier (PPF) of two different representations of the firm's production technology, across two-states of nature (tomorrow) and for two different input K levels (chosen today). The left panel shows the PPF (black solid line) of a smooth (differentiable) representation of the technology. The firm is assumed to be producing at point A and the slope of the PPF at point A (M_1) identifies the equilibrium marginal rate of transformation. The right panel shows the PPF of a standard representation of the technology implied by a standard production function such as $Y(s) = \epsilon(s)F(K)$, in which K is the input level, $Y(s)$ and $\epsilon(s)$ are tomorrow's output and state contingent productivity level respectively, both a function of tomorrow's state of nature $s = 1, 2$. In this representation, the PPF is not smooth across states of nature and so the marginal rate of transformation is not well defined, as shown by the two possible slopes of the PPF at the production point A (M_1 and M_2). In both representations, firms can substitute output over time by changing the input level K. An increase in the input level today will move production from point A to point B, thus increasing production in both states of nature tomorrow.

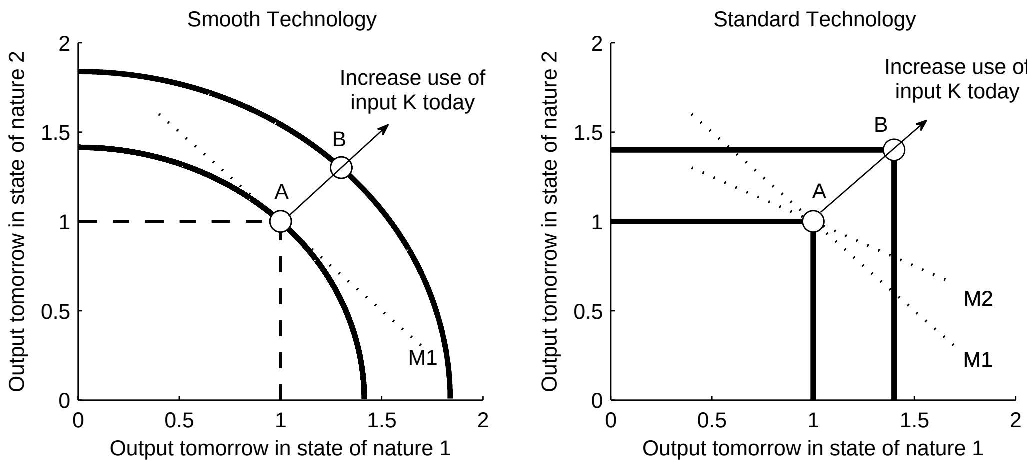

Figure 2
Pricing Errors

The figure shows the plot of the predicted versus realized excess returns per annum implied by the estimation of the one common productivity factor specification of the production-based model on the following portfolio sorts: (i) top-left: 6 size and book to market portfolios; (ii) top-right: 9 risk double sorted portfolios; (iii) bottom-left: 5 industry portfolios and (iv) bottom-right: all the 20 portfolios together. In the figure, each digit number represents one portfolio. In the size and book to market portfolios, the first digit refers to the size sort (1 and 2 for small and big respectively), and the second digit refers to book to market sort (1, 2 and 3 indicating the growth, neutral and value portfolios respectively). In the risk portfolios, the first digit refers to the pre-ranking output beta sort and the second digit refers to the pre-ranking price beta sort (1, 2 and 3 indicating the low, medium and high pre-ranking beta of the corresponding factor respectively). The data are annual and the sample is $1930 - 2007$.

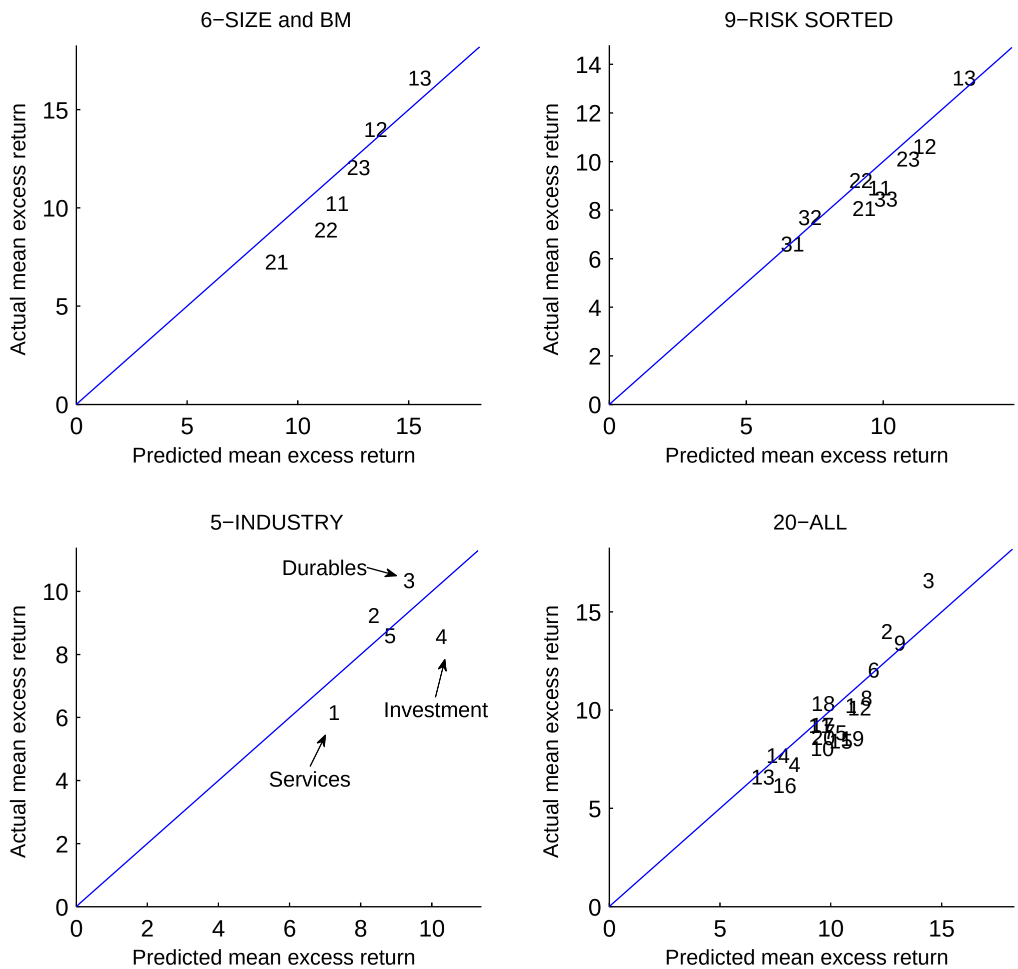

Figure 3
Estimated Common Productivity Factor and Marginal Rate of Transformation

This Figure plots the time series of the estimated common productivity factor (F_t) and the estimated log marginal rate of transformation (MRT) of the one common productivity factor specification of the production-based model. All series are estimated based on the first stage GMM estimation of the production-based model on the Fama-French 6 portfolios sorted on size and book-to-market and are demeaned. Shaded bars are NBER recession years. The data are annual and the sample is 1930 − 2007.

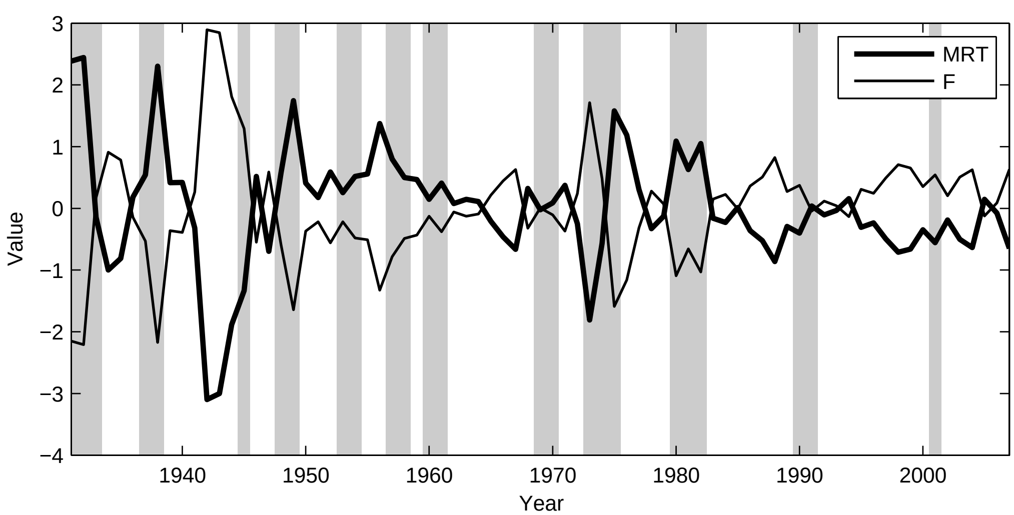

Figure 4
Estimated Marginal Rate of Transformation and Intertemporal Marginal Rate of Substitution

This Figure plots the time series of the estimated log marginal rate of transformation of the one common productivity factor specification of the production-based model and the estimated log intertemporal marginal rate of substitution of the Yogo (2006) durable consumption model. All series are estimated based on the first stage GMM estimation of each model on the Fama-French 6 portfolios sorted on size and book-to-market and are demeaned. Shaded bars are NBER recession years. The data are annual and the sample is $1930 - 2007$.

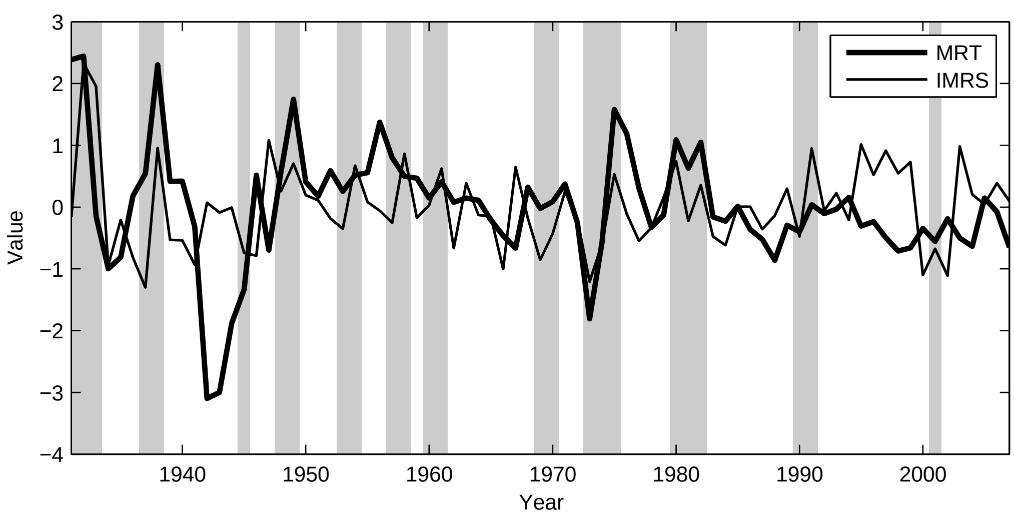

Figure 5
Out of Sample Pricing Errors

The figure shows the plot of the out of sample predicted versus realized excess returns (per annum) for portfolios of two asset classes: (i) 5 bond portfolios (Bond); and (ii) 8 currency portfolios (Currency), from Lustig and Verdelhan (2006). The predicted excess returns of each portfolio are computed from $\mathbb{E}[R_t^e] = -\text{Cov}(M_t, R_t^e)$, in which R_t^e is a vector with the excess returns of the set of portfolios and M_t is the estimated marginal rate of transformation of the one common productivity factor specification of the production-based model on the Fama-French 6 portfolios sorted on size and book-to-market. In the figure, each digit number represents one portfolio. The data are annual and the sample is $1953-2002$ for the currency portfolios and $1952-2007$ for the bond portfolios.

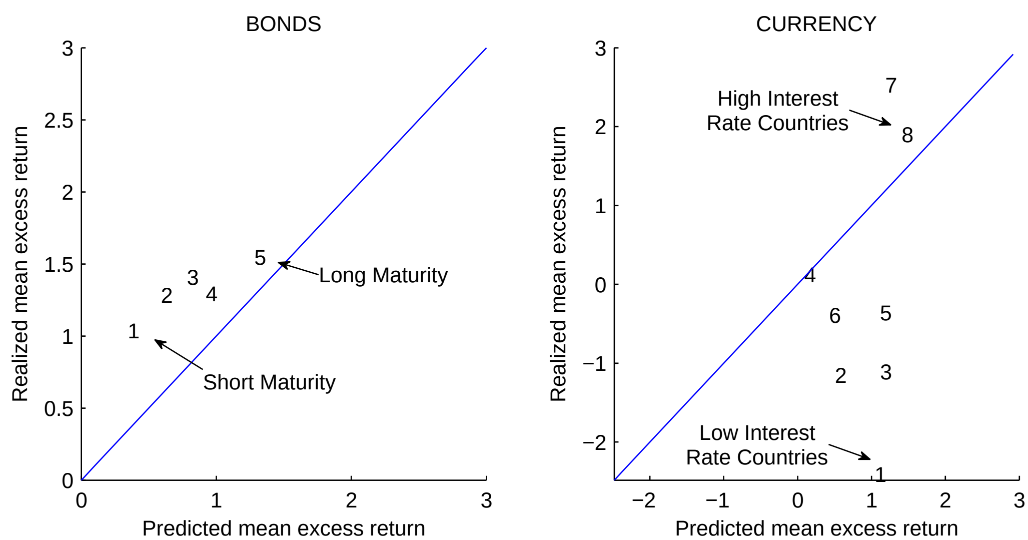

Figure 6

Predicted Conditional Equity Premium and Realized Dividend Yield

The figure shows the time series of the predicted equity premium (EP) implied by the production-based model and the time series of the realized dividend yield in the US economy. The predicted equity premium is obtained from equation (18) and by specifying an AR(1)-GARCH(1,1) process for the estimated demeaned log marginal rate of transformation. The data are annual and the sample is $1930 - 2007$.