Asset Prices and Risk Sharing in Open Economies

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Abstract

This paper proposes a two-good, two-country general equilibrium model with external habits and home-biased preferences that addresses a number of international finance puzzles. Specifically, the model reconciles the high degree of international risk sharing implied by relatively smooth exchange rates with the modest cross-country consumption growth correlations seen in the data, resolving the Brandt, Cochrane and Santa-Clara (2006) puzzle. Furthermore, the model matches the empirically observed low correlation between exchange rate changes and international consumption growth rate differentials. For both effects, the fundamental mechanism is time variation in consumption growth volatility, which is endogenously generated through international trade. Asset prices depend on a weighted average of the two countries' time-varying risk aversion, with the weights determined by the wealth and degree of home bias of each country. Simulation results indicate that the model is successful in matching key empirical exchange rate and international trade moments, as well as the standard asset pricing moments.

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1 Introduction

This article presents a model which highlights, in a tractable way, the links between asset prices, exchange rates and international risk sharing generated by international trade in goods and assets. The model proposes a solution to a number of international finance puzzles that are related to the connection between the aforementioned three economic concepts, with the primary focus being on the international risk sharing puzzle. Furthermore, the model clearly illustrates how international trade affects equity prices and risk-free rates vis-à-vis the closed economy benchmark and explicitly connects real exchange rates and equity prices.

The international risk sharing puzzle, illustrated in detail in Brandt, Cochrane and Santa-Clara (2006), is the apparent disconnect between relatively modest empirical cross-country consumption growth rate correlations and the extremely high degree of international risk sharing implied by the relatively low real exchange rate volatility observed in the data.[1] If financial markets are complete, the key relationship that generates the links between asset and currency prices and risk sharing is the no-arbitrage relationship

$$\frac{M^*_{t+1}}{M_{t+1}} = \frac{E_{t+1}}{E_t} \tag{1}$$

where M_{t+1} and M^*_{t+1} are the home and foreign stochastic discount factor (SDF), respectively, and E_t is the real exchange rate (domestic price of foreign currency, in real terms, i.e. an increase in E_t denotes a real depreciation of the domestic currency). This relationship is not without assumptions: it holds only when financial markets are frictionless, in the sense that investors in each country can freely invest in assets denominated in any of the two currencies.[2] Perfect risk sharing between the two countries means $M^*_{t+1} = M_{t+1}$, which implies a constant real exchange rate.[3]

Taking logs and then unconditional variances on each side of (1), we get

$$var(m^*_{t+1}) + var(m_{t+1}) - 2\sigma(m_{t+1})\sigma(m^*_{t+1})\rho(m_{t+1}, m^*_{t+1}) = var(\Delta e_{t+1})$$

[1] As will be discussed later, real exchange rate volatility is low only compared to asset return volatility; it is quite high compared to macroeconomic (income or consumption) volatility.

[2] Complete markets are not necessary for (1) to hold. In the presence of market incompleteness, (1) holds with M_{t+1} being the (unique) projection of all (i.e. domestic and foreign) investors' intertemporal marginal rate of substitution (IMRS), expressed in domestic currency units, on X_{t+1}, and M^*_{t+1} the (unique) projection of all investors' IMRS, expressed in foreign currency units, on X^*_{t+1}, where X_{t+1} (X^*_{t+1}) is the space spanned by the domestic (foreign) currency returns of all assets, domestic and foreign. See Backus, Foresi and Telmer (2001) and Brandt, Cochrane and Santa-Clara (2006).

[3] In the international macroeconomics and finance literature, optimal risk sharing is sometimes defined as being equivalent to the achievement of a Pareto optimal allocation. In that case, (1) is the optimal risk sharing condition; for special cases of this condition, see, for example, Cole and Obstfeld (1991), Backus and Smith (1993), Lewis (1996) and Obstfeld and Rogoff (2000). This paper, following Brandt et el. (2006), uses the term "perfect risk sharing" to refer to the more stringent condition $M_{t+1} = M^*_{t+1}$; the reason is that, as explained in detail in Brandt et al. (2006) and in this paper, the international risk sharing puzzle regards SDF correlations, not Pareto optimality.

with small letters denoting the log of their capital letter counterpart. Using the logic of Hansen and Jagannathan (1991) volatility bounds, the high Sharpe ratios we observe in asset markets imply very high pricing kernel volatility. Unless the correlation between the two pricing kernels is extremely high, high kernel volatility cannot be reconciled with the empirically observed modest levels of real exchange rate volatility. Setting, for example, $\sigma(m_{t+1}) = \sigma(m^*_{t+1}) = 50\%$ and considering $\sigma(\Delta e_{t+1}) = 10\%$ (in line with empirical real exchange rate volatility for major currency pairs), we can easily see that prices imply that $\rho(m_{t+1}, m^*_{t+1}) = 0.98$. Under CRRA preferences, $m_{t+1} = \log \beta - \gamma \Delta c_{t+1}$ and $m^*_{t+1} = \log \beta - \gamma \Delta c^*_{t+1}$, so $\rho(m_{t+1}, m^*_{t+1}) = \rho(\Delta c_{t+1}, \Delta c^*_{t+1})$. Then, to square prices with quantities, we need $\rho(\Delta c_{t+1}, \Delta c^*_{t+1}) = 0.98$; unfortunately, the observed cross-country consumption growth correlations are typically much lower, with correlations of 0.9 and above not being even remotely plausible. The only way we can have e.g. $\rho(m_{t+1}, m^*_{t+1}) = 0.3$ (in line with empirical cross-country consumption growth correlations) is if $\sigma(\Delta e_{t+1}) = 65\%$; as mentioned above, this is highly counterfactual. This, in a nutshell, is the puzzle: prices tell us that risk is nearly perfectly shared among countries, but quantities tell us otherwise.[4] Consequently, any model that aims to explain the relationship between asset returns and exchange rates should address international risk sharing, reconciling high unconditional pricing kernel correlations with relatively modest unconditional consumption growth rate correlations.

A related puzzle to be addressed is the exchange rate disconnect puzzle, illustrated by Backus and Smith (1993). Starting with (1), it is easy to see that for CRRA preferences we get

$$corr(\Delta c_{t+1} - \Delta c^*_{t+1}, \Delta e_{t+1}) = 1 \tag{2}$$

irrespective of the value of γ. Backus and Smith (1993) derive this result in a more general setting. However, in the data, consumption growth rate differentials appear to be decoupled from real exchange rate changes; using data from 8 OECD countries, Backus and Smith (1993) show that the average correlation between per capita consumption growth rate differentials and real exchange rate changes is -0.056, with a range of [-0.63, 0.21], a far cry from the theoretical value of 1.

Another issue in international macroeconomics is the "remarkable", in the words of Obstfeld and Rogoff (2000), volatility of real exchange rates. As mentioned earlier, from the perspective of asset pricing, real exchange rate volatility is relatively small: asset return volatility is around $15\% - 20\%$ per year, while pricing kernel volatility is even higher, of the order of 50%. However, from the perspective of international macroeconomics, the volatility of real exchange rate changes should not be far from consumption or income growth volatility, around $1\% - 3\%$ per year; it is, instead, almost an order of magnitude higher.

[4] Brandt et al. (2006) measure risk sharing using a risk sharing index, which also detects differences in *scale*. Since perfect risk sharing is the equalization of the two countries' pricing kernels, $m_{t+1} = 2m^*_{t+1}$ does not imply perfect risk sharing, although $\rho(m_{t+1}, m^*_{t+1}) = 1$. Therefore, high correlation is not sufficient for high risk sharing; it is, however, necessary.

This paper proposes a two-country endowment model that incorporates external habits and consumption home bias in preferences. In the model, the global economy is comprised of two countries, each represented by a stand-in agent endowed with a stream of a differentiated perishable good. Each of the two agents has Menzly, Santos and Veronesi (2004) external habit preferences, with the habit defined on a home-biased, CES aggregated consumption basket of the two goods. This model leads to an economically intuitive solution to both the international risk sharing puzzle and the exchange rate disconnect puzzle.

Regarding the international risk sharing puzzle, the model implies that countries indeed share risk to a very large degree through trade in goods and assets. Specifically, trade generates endogenous time variation in consumption growth volatility: the conditionally relatively less risk averse country assumes more of the global endowment risk. In other words, the conditionally more risk averse country has low consumption risk, while the less risk averse country has high consumption risk. On the other hand, the conditionally less risk averse country has low conditional sensitivity to consumption growth risk, while the more risk averse country is very sensitive to consumption risk. To understand how this resolves the puzzle, consider the market price of consumption risk. Since, for each country, the conditional market price of consumption risk is an increasing function of both conditional risk aversion and conditional consumption growth volatility, the two effects (volatility and sensitivity) push the *relative* market price of risk of the two countries to different directions. However, since the magnitude of the two effects is almost equal, those two effects, combined, balance each other, leading to a very high cross-country correlation of market prices of risk and, thus, pricing kernels. However, cross-country consumption growth correlation is modest, since there is no sensitivity effect to counter the volatility effect.

Regarding the exchange rate disconnect puzzle, habits decouple marginal utility growth from consumption growth. This effect generates very low correlation between consumption growth rate differentials and real exchange rate changes, despite the fact that the correlation between pricing kernel differentials and real exchange rate changes is, by construction, perfect.

The model also sheds light on the issue of asset pricing in open economies. Specifically, the model generates an economically intuitive solution for the price of the two countries' total wealth portfolios: the price-dividend ratio of each total wealth portfolio is determined by a weighted average of the two countries' time-varying relative risk aversion coefficients, with the weights depending on the initial wealth and the degree of home bias of the two countries. When the economy is closed, the solution collapses to the Menzly, Santos and Veronesi (2004) pricing results, so the model clearly illustrates how international trade affects asset prices and returns. The connection of asset prices with exchange rates is also straightforward: the real exchange rate is a function of *both* the endowment ratio *and* the price-dividend ratios of the two total wealth portfolios. Thus, real exchange rate volatility is generated by two economic mechanisms: time variation in relative endowments and time variation in price-dividend ratios. The latter, asset pricing-related, mechanism amplifies the effects of the former, endowment-related, mechanism, so real exchange rate changes are much more volatile than endowment growth rates. The failure of most standard international macroeconomic models to generate substantial real exchange rate volatility can, thus, be traced to their inability to generate time-varying asset price-dividend ratios.

This paper is part of the recent literature that focuses on the connections between asset prices and exchange rates. The model in this paper builds on Pavlova and Rigobon (2007). They use a Lucas (1982) two-country, two-good model to examine the effects of the terms of trade on asset prices and exchange rates when preferences are characterized by demand shocks. Inter alia, they use financial data to extract latent factors implied by their model and show that those factors can be used to predict macroeconomic variables and ameliorate puzzles arising in the international real business cycle literature. Despite its success in addressing macroeconomic questions, the ability of their model to match asset prices and returns is limited by its inability to generate time-varying asset price-dividend ratios.[5] Other international asset pricing models that focus on terms of trade effects are Cole and Obstfeld (1991), Zapatero (1995) and Serrat (2001).

Recent papers extend standard asset pricing models to examine international finance issues. Regarding habits, Verdelhan (2008a) uses a two-country, one-good model in which each country has an exogenously specified i.i.d. consumption growth process and Campbell and Cochrane (1999) external habit preferences. The model is able to explain the forward premium puzzle, but generates real exchange rates that are both highly volatile, implying poor international risk sharing, and excessively linked to consumption growth. Verdelhan (2008b), a companion paper, assumes i.i.d. endowment growth and allows for international trade characterized by proportional and quadratic trade costs, thus endogenizing consumption; the ability to share risk by international trade lowers real exchange rate volatility to realistic levels, but the real exchange rate remains very closely linked to consumption growth, so the Backus and Smith (1993) puzzle cannot be resolved.[6] Bekaert (1996) examines currency risk premia using a two-country monetary model which features durability and habit persistence. Moore and Roche (2006) embed Campbell and Cochrane (1999) preferences with "deep" habits (Ravn, Schmitt-Grohe and Uribe (2006)) in a flexible-price monetary model in order to address both the exchange rate disconnect puzzle and the forward premium puzzle. Shore and White (2006) address the portfolio home bias puzzle with a model that incorporates external habit formation. Aydemir (2008) uses a two-country, one-good external habits model in order to examine international equity market return correlations.

[5] To be precise, price-dividend ratios are non-stochastic. Specifically, when the time horizon is finite, the price-dividend ratio of each country's total wealth portfolio is a deterministic function of time. When the time horizon is infinite, the price-dividend ratio is constant.

[6] The author suggests that this result may be caused by the one-good assumption. It should be noted that Verdelhan (2008b) does not examine international consumption growth correlations, so the paper does not explore the international risk sharing puzzle.

Colacito and Croce (2008a) utilize the Bansal and Yaron (2004) long-run risks framework in order to address the international risk sharing puzzle. They consider a two-country, two-good closed economy endowment model in which each country has Epstein and Zin (1989) preferences and an exogenously specified consumption growth process featuring a slow moving, predictable component. They show that the puzzle can be resolved if the two predictable components, the domestic and the foreign one, are highly correlated. In Colacito and Croce (2008b), they extend their model to open economies, allowing for international trade and endogenizing consumption, in order to revisit the Cole and Obstfeld (1991) results; they show that international portfolio diversification may produce significant welfare gains in the presence of long-run risk.[7] Bansal and Shaliastovich (2007) also use a long-run risks model in order to, inter alia, address the forward premium puzzle.

Farhi and Gabaix (2008) propose a two-country rare disasters model to explain the cross-country joint dynamics of exchange rates, bonds, stocks and options. Lustig and Verdelhan (2006) use the Yogo (2006) model in order to empirically illustrate the effects of consumption growth risk on currency risk premia.

One of the key assumptions of this paper is external habit formation. Habits, internal or external, have been used in much of the recent asset pricing literature.[8] The present paper postulates Menzly et al. (2004) external habits, which share the motivation of Campbell and Cochrane (1999) habits, but model the inverse surplus consumption ratio. Buraschi and Jiltsov (2007) use the same mean-reverting process for the inverse surplus consumption ratio in order to study the term structure of interest rates.[9]

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 presents the equilibrium macroeconomic prices and quantities and explains how the international risk sharing puzzle is resolved. Section 4 explores the asset pricing implications of the model. Section 5 reports the simulation results. Section 6 concludes. The Appendix contains the proofs and all supplementary material not included in the main body of the paper.

[7] As in Colacito and Croce (2007), matching the empirical volatility level of real exchange rate changes requires that long-run endowment shocks be highly internationally correlated.

[8] See, for example, Sundaresan (1989), Abel (1990), Constantinides (1990), Detemple and Zapatero (1991), Ferson and Constantinides (1991), Heaton (1995), Jermann (1998), Boldrin, Christiano and Fisher (2001) and Chan and Kogan (2002).

[9] Santos and Veronesi (2006) use a similar formulation to examine the cross section of stock returns. Bekaert, Engstrom and Grenadier (2005) also model the inverse surplus consumption ratio: in their model, it is a mean-reverting process driven by two shocks, a consumption growth shock and an exogenous shock in risk appetite (or "mood").

2 The model

2.1 The structure of the economy

The world economy is comprised of two countries, Domestic and Foreign, each of which is populated by a single risk-averse representative agent who receives an endowment stream of a single differentiated perishable good: the domestic agent is endowed with the domestic good, while the foreign agent is endowed with the foreign good. Economic activity takes place in the time interval $[0,\infty)$. Uncertainty in this economy is represented by a filtered probability space $(\Omega, \mathcal{F}, \mathbf{F}, P)$, where $\mathbf{F}=\{\mathcal{F}_t\}_{t\in[0,\infty)}$ is the filtration generated by the two-dimensional Brownian motion $\mathbf{B} = [B^X, B^Y]'$, augmented by the null sets. All stochastic processes introduced in the remainder of the paper are assumed to be progressively measurable with respect to $\mathbf{F}$ and to satisfy all the necessary regularity conditions for them to be well-defined. All (in)equalities that involve random variables hold P-almost surely.

The endowment sequence of the domestic good is denoted by $\{\widetilde{X}_t\}$ and that of the foreign good by $\{\widetilde{Y}_t\}$. Both processes are assumed to be of the form:

$$d\log\widetilde{X}_t = \mu_t^X dt + \sigma^X dB_t^X \tag{3}$$

and

$$d\log\widetilde{Y}_t = \mu_t^Y dt + \sigma^Y dB_t^Y \tag{4}$$

Note that both drifts are left unspecified. On the other hand, I specify that endowment growth is homoskedastic for both countries. This is a key point: any conditional heteroskedasticity arising in this model is endogenously generated.[10] The two endowment shocks dB_t^X and dB_t^Y are correlated with instantaneous correlation ρ^{XY}. Thus, the instantaneous covariance matrix of $\mathbf{dB}_t$ is

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & \rho^{XY} \\ \rho^{XY} & 1 \end{bmatrix}$$

Both goods are frictionlessly traded internationally, so the price of each good (in units of the numeraire good) is the same in both countries (law of one price). Denote by Q and Q^* the price of the domestic good and the foreign good, respectively, in terms of the numeraire. Since this is a non-monetary economy, only relative prices are determined; without loss of generality, the domestic good is considered the numeraire good, so $Q_t \equiv 1, \forall t \in [0,\infty)$. Then, $Q^* = \frac{Q^*}{Q}$ denotes the terms of trade (the ratio of the price of exports over the price of imports) for the foreign country, which are the inverse terms of trade for the domestic country; in the remainder of the paper, Q^* will be called terms of trade without further specification.[11]

[10] This specification is adopted for simplicity; the extension to arbitrary diffusion processes σ_t^X and σ_t^Y is trivial.

[11] This definition of the terms of trade (price of exports over the price of imports) is the one used in international trade. In international macroeconomics, sometimes the inverse definition is applied - see, for example, Chapter

Finally, financial markets are complete and there are no frictions in the international trade of financial assets, so the no arbitrage condition (1) holds $\forall t \in [0, \infty)$.

2.2 Preferences

The domestic representative agent maximizes expected discounted utility

$$E_0\left[\int_0^\infty e^{-\rho t} u(X_t, Y_t) dt\right]$$

where $\rho > 0$ is her subjective discount rate, and her instantaneous utility function is

$$u(X_t, Y_t) = \log(X_t^a Y_t^{1-a} - H_t) = \log(C_t - H_t) \tag{5}$$

where X_t and Y_t is the quantity of the domestic and foreign good, respectively, she consumes at time t, $C \equiv X^a Y^{1-a}$ is the domestic consumption basket and H_t is the time t habit level associated with that consumption basket.

Two main assumptions about the domestic agent's preferences are adopted here. The first assumption is that the domestic consumption basket is a Cobb-Douglas aggregate of the two goods. Then, the elasticity of substitution between the two goods is unity, so the goods are imperfect substitutes. A second implication is that the domestic agent may exhibit home bias, in the sense that her preferences over the two goods may not be necessarily symmetric. Parameter $a \in [0, 1]$ denotes the degree of relative preference for the domestic good. When $a > 0.5$, the agent is home biased: one unit of the domestic good provides her with more utility than one unit of the foreign good. When $a = 1$, the agent is completely home biased: she only gets utility from the domestic good, so no international trade occurs in equilibrium. When $a = 0.5$, the agent has symmetric preferences towards the two goods, so no home bias exists.

The second main assumption regarding preferences is the existence of an external habit. It should be noted that the habit is over the *consumption basket* and not over *individual goods' consumption*. This specification is in line with the standard asset pricing literature: although asset pricing models usually assume a single good, empirically this good is taken to be aggregate consumption, which consists of many goods. I further assume that the external habit is of the Menzly et al. (2004) form. Specifically, it is assumed that the inverse surplus consumption ratio $G = \left(\frac{C-H}{C}\right)^{-1}$ solves the stochastic differential equation

$$dG_t = k\left(\bar{G} - G_t\right) dt - \delta\left(G_t - l\right)\left(\frac{dC_t}{C_t} - E_t\left(\frac{dC_t}{C_t}\right)\right) \tag{6}$$

The inverse surplus consumption ratio G is a mean-reverting process, reverting to its long-

11 in Cooley (1995).

run mean of $\bar{G}$ at speed $k > 0$ and is driven by consumption growth shocks. The parameter $\delta > 0$ scales the impact of the consumption growth shock and the parameter $l \geq 1$ is the lower bound of the inverse surplus consumption ratio. Obviously, $\bar{G} > l$.[12] The local curvature of the utility function is $-\frac{u_{CC}(C,H)}{u_C(C,H)}C = G$; for that reason, and in a slight abuse of terminology, in the rest of the paper I will refer to G as domestic risk aversion.

The preferences of the foreign stand-in agent are similar. Her instantaneous utility function is

$$u^*(X_t^*, Y_t^*) = \log\left((X_t^*)^{a^*}(Y_t^*)^{1-a^*} - H_t^*\right) = \log(C_t^* - H_t^*) \tag{7}$$

where X_t^* and Y_t^* is the agent's time t consumption of the domestic and foreign good, respectively, $C^* \equiv (X^*)^{a^*}(Y^*)^{1-a^*}$ is the foreign consumption basket and H_t^* is the foreign habit level at time t. Note that home consumption bias for the foreign agent implies $a^* < 0.5$.

The results discussed in the remainder of the paper refer to non-boundary parameter values $a \in (0,1)$ and $a^* \in (0,1)$, unless otherwise noted. Furthermore, the empirically relevant case is $a^* < 0.5 < a$, with both countries exhibiting home bias. However, the weaker condition $a^* < a$ suffices for the qualitative characterization of the results in this paper.[13] Thus, when discussing the results, I will focus on the case $0 < a^* < a < 1$. The difference in the preferences for the domestic good $a - a^*$ will be called the degree of home bias.

The foreign agent also has external habits, with her inverse surplus consumption ratio process satisfying:

$$dG_t^* = k\left(\bar{G} - G_t^*\right)dt - \delta\left(G_t^* - l\right)\left(\frac{dC_t^*}{C_t^*} - E_t\left(\frac{dC_t^*}{C_t^*}\right)\right) \tag{8}$$

For simplicity, it is assumed that the preference parameters k, δ, $\bar{G}$ and l are the same in both countries.[14]

[12] The Menzly et al. (2004) model shares many of the properties of the Campbell and Cochrane (1999) model, which assumes a specification for the process of the surplus consumption ratio $S_t = \frac{C_t - H_t}{C_t}$. In the Campbell and Cochrane (1999) model, the support of S is $(0, \bar{S}]$. In the Menzly et al. (2004) model the support of G is $[l, \infty)$, so S is bounded in $(0, \frac{1}{l}]$; the support of S is the same for $l = \frac{1}{\bar{S}}$. However, the two models are not isomorphic: for example, see Hansen (2008) for a discussion of their differing implications for long-run returns.

[13] Under that weaker condition, each country cares more about its good than the other country does, but does not necessarily care more about its own good than about the other country's good; the latter requires the stronger condition $a^* < 0.5 < a$. Condition $a^* < a$ can be called *relative* home bias, while condition $a^* < 0.5 < a$ can be called *absolute* home bias. In the remainder of the paper, the term *home bias* will be used to refer to relative home bias.

[14] This assumption is made for convenience and can be easily relaxed without any qualitative difference in the results.

2.3 Prices and exchange rates

Given that the domestic consumption basket is $C = X^a Y^{1-a}$, the associated time t price index is:

$$P_t = \left(\frac{Q_t}{a}\right)^a \left(\frac{Q_t^*}{1-a}\right)^{1-a} \tag{9}$$

P_t is the time t price of one unit of domestic consumption in units of the numeraire good; it is defined as the minimum expenditure required to buy a unit of the domestic consumption basket C and is derived by minimizing the relevant expenditure function.

Similarly, the foreign price index is:

$$P_t^* = \left(\frac{Q_t}{a^*}\right)^{a^*} \left(\frac{Q_t^*}{1-a^*}\right)^{1-a^*}$$

which is the price, in terms of the numeraire good, of one unit of the foreign consumption basket.

Therefore, the time t real exchange rate, which expresses the price of a unit of the foreign consumption basket in units of the domestic consumption basket, is:

$$E_t = \frac{P_t^*}{P_t} = \frac{a^a(1-a)^{1-a}}{(a^*)^{a^*}(1-a^*)^{1-a^*}} (Q_t^*)^{a-a^*} \tag{10}$$

using the fact that $Q_t = 1$, $\forall t \in [0, \infty)$. Trivially, when the preferences of the two countries are identical ($a = a^*$), the two consumption baskets are also identical ($C = C^*$). Then, since the absence of trade frictions implies the law of one price, the price of the two baskets is the same, and the real exchange rate is constant at 1 (Purchasing Power Parity). This is the case of perfect risk sharing: the absence of market frictions allows agents with identical preferences to fully share risk. In a frictionless world, what generates real exchange rate volatility is the difference in the two countries' preferences, and thus the fact that the two consumption baskets are not identical: $C \neq C^*$. Then, volatility in the terms of trade Q^* generates variation in the relative price of the two consumption baskets. In fact, in this model, due to the assumption of unit elasticity of substitution between the two goods, real exchange rate change volatility is proportional to the degree of home bias $a - a^*$.

3 Equilibrium prices and quantities

3.1 The planner's problem

Under the assumption of market completeness, the competitive equilibrium (CE) allocation is equivalent to a central planner's allocation, with the planner taking the laws of motion for G and

G^* as given.[15] For the CE solution to be identical to the planner's problem solution, the welfare weights must be determined endogenously.[16] We will see in a later section that the appropriate welfare weights can be easily calculated in this model, so we can first solve the planner's problem and then calculate the welfare weights that equate the planner's problem equilibrium with the CE.

The social planner maximizes a weighted average of the two countries' expected utility, with welfare weights being λ and $\lambda^* = 1 - \lambda$, for the domestic and foreign country, respectively:

$$\max_{\{X_t, Y_t, X_t^*, Y_t^*\}} E_0 \left[\int_0^\infty e^{-\rho t} \left(\lambda \log(C_t - H_t) + \lambda^* \log(C_t^* - H_t^*) \right) dt \right] \tag{11}$$

subject to the resource constraints $X_t + X_t^* = \widetilde{X}_t$ and $Y_t + Y_t^* = \widetilde{Y}_t$.

3.2 Consumption

Solving the planner's problem (see Appendix, section A.1), we get the equilibrium consumption allocation. For the home agent:

$$X_t = \omega_t \widetilde{X}_t, \;\; Y_t = \omega_t^* \widetilde{Y}_t \tag{12}$$

and for the foreign agent:

$$X_t^* = (1 - \omega_t) \widetilde{X}_t, \quad Y_t^* = (1 - \omega_t^*) \widetilde{Y}_t \tag{13}$$

where I introduce the *share functions* ω_t and ω_t^*

$$\omega_t = \omega \left(\frac{G_t^*}{G_t} \right) \equiv \frac{a\lambda}{a\lambda + a^* \lambda^* \left(\frac{G_t^*}{G_t} \right)}$$

$$\omega_t^* = \omega^* \left(\frac{G_t^*}{G_t} \right) \equiv \frac{(1-a)\lambda}{(1-a)\lambda + (1-a^*)\lambda^* \left(\frac{G_t^*}{G_t} \right)}$$

with ω_t (ω_t^*) being the proportion of domestic (foreign) endowment consumed by the domestic agent. In the case of complete home bias ($a = 1$, $a^* = 0$), it can easily be shown that $\omega_t = 1$ and $\omega_t^* = 0$, $\forall t \in [0, \infty)$: each country consumes its endowment, so no trade occurs and both economies are closed in equilibrium. Both share functions are decreasing in the risk aversion ratio $\frac{G_t^*}{G_t}$. It should be noted that under home bias ($a > a^*$), ω_t^* is more sensitive than ω_t to the risk aversion ratio $\frac{G_t^*}{G_t}$, so $\frac{\omega_t}{\omega_t^*}$ is increasing in $\frac{G_t^*}{G_t}$.

[15] For the planner's solution to coincide with the CE solution, the planner has to take into account the externality arising from external habit formation. Thus, the CE solution will not be uncontrained Pareto optimal, but constrained Pareto optimal, with the constraint being the assumed external habit processes.

[16] Specifically, welfare weights are related to the intertemporal budget constraints of the two countries.

Therefore, domestic consumption is

$$C_t = \omega_t^a (\omega_t^*)^{1-a} \widetilde{X}_t^a \widetilde{Y}_t^{1-a} \tag{14}$$

and foreign consumption is

$$C_t^* = (1-\omega_t)^{a^*} (1-\omega_t^*)^{1-a^*} \widetilde{X}_t^{a^*} \widetilde{Y}_t^{1-a^*} \tag{15}$$

Consumption depends on three state variables: the two endowment levels $\widetilde{X}_t$ and $\widetilde{Y}_t$, and the ratio of risk aversions $\frac{G_t^*}{G_t}$. The effects of endowment levels on consumption are straightforward: since both countries consume both goods, both domestic and foreign consumption are increasing in both endowments. However, the degree that each country's consumption is affected by each endowment's fluctuations depends on relative preferences: unsurprisingly, under home-biased preferences ($a > a^*$), each country's consumption is more sensitive to its own endowment than to the other country's endowment.

More interesting are the effects of the ratio $\frac{G_t^*}{G_t}$, through the share functions: domestic consumption (X_t, Y_t and so C_t) is decreasing in $\frac{G_t^*}{G_t}$, while foreign consumption is increasing in $\frac{G_t^*}{G_t}$. When the foreign agent becomes *relatively* more risk averse than the domestic agent, consumption is shifted from the domestic country to the foreign country, and vice versa. This is an international risk sharing effect: each period, consumption flows to the country that needs it the most, i.e. the country which is closer to its habit and is more averse to further consumption reduction.

Since consumption and habit level are jointly determined, understanding the evolution of domestic and foreign consumption over time requires explicitly solving for the two equilibrium consumption processes as functions of the two exogenous shocks dB_t^Y and dB_t^Y. The following proposition, the proof of which can be found in the Appendix, presents the result.[17]

Proposition 1 *The equilibrium consumption process for the domestic representative agent is*

$$\frac{dC_t}{C_t} - E_t\left(\frac{dC_t}{C_t}\right) = \boldsymbol{\sigma}_t^{C\prime}\mathbf{dB}_t = \sigma_t^{CX} dB_t^X + \sigma_t^{CY} dB_t^Y$$

with

$$\sigma_t^{CX} = \frac{1}{D_t^c}\left(a + (ak_t^* + a^*k_t)\,\delta\left(\frac{G_t^* - l}{G_t^*}\right)\right)\sigma^X \quad \textit{and} \tag{16}$$

$$\sigma_t^{CY} = \frac{1}{D_t^c}\left((1-a) + ((1-a)k_t^* + (1-a^*)k_t)\,\delta\left(\frac{G_t^* - l}{G_t^*}\right)\right)\sigma^Y \tag{17}$$

[17] Proposition 1 focuses on the diffusion terms of the two consumption processes; the two drift terms depend on endowment drifts, which have been left unspecified. The simulation section of the paper considers specific endowment growth drift specifications and their results for the mean of consumption growth rates.

and the equilibrium consumption process for the foreign representative agent is

$$\frac{dC_t^*}{C_t^*} - E_t\left(\frac{dC_t^*}{C_t^*}\right) = \boldsymbol{\sigma}_t^{C^*\prime}\mathbf{dB}_t = \sigma_t^{C^*X} dB_t^X + \sigma_t^{C^*Y} dB_t^Y$$

with

$$\sigma_t^{C^*X} = \frac{1}{D_t^c}\left(a^* + (ak_t^* + a^*k_t)\,\delta\left(\frac{G_t - l}{G_t}\right)\right)\sigma^X \quad \text{and} \tag{18}$$

$$\sigma_t^{C^*Y} = \frac{1}{D_t^c}\left((1-a^*) + ((1-a)k_t^* + (1-a^*)k_t)\,\delta\left(\frac{G_t - l}{G_t}\right)\right)\sigma^Y \tag{19}$$

where k_t, k_t^* and D_t^c are functions of G_t and G_t^* defined in the Appendix (equations (55), (56) and (57), respectively). For $0 < a^* < a < 1$, it holds that $0 < k_t < 1$ and $0 < k_t^* < 1$ and $D_t > 0$, $\forall t \in [0, \infty)$.

The key result is that both consumption growth processes have time-varying volatility, even though both endowment growth processes are homoskedastic. Note that σ_t^{CX} and σ_t^{CY} are (roughly) proportional to $\frac{G_t^* - l}{G_t^*}$; thus, domestic conditional consumption growth volatility is roughly scaled by $\frac{G_t^* - l}{G_t^*}$. Conversely, foreign conditional consumption growth volatility is scaled by $\frac{G_t - l}{G_t}$. Thus, each country's conditional consumption growth volatility is increasing in the *other* country's conditional risk aversion. This, again, is the result of risk sharing: the *conditionally* less risk averse country insures the more risk averse country by assuming more of the global endowment risk. This way, international trade in goods and assets allows countries to allocate endowment risk efficiently.

To examine the impact of habit preferences, consider the log utility economy, to which the model economy reduces in the absence of external habit formation. In that case, consumption growth is homoskedastic:

$$\sigma_t^{CX} = a\sigma^X,\ \sigma_t^{CY} = (1-a)\sigma^Y$$

and

$$\sigma_t^{C^*X} = a^*\sigma^X, \quad \sigma_t^{C^*Y} = (1-a^*)\sigma^Y$$

Habit preferences lead countries to share risk through the reallocation of consumption growth risk; standard log (and, in general, CRRA) preferences do not.

Since this is a complete markets setting, the aforementioned risk reallocation occurs through transactions in Arrow-Debreu securities. International risk sharing, in this context, means that, at each period, the conditionally more risk averse country holds the Arrow-Debreu consumption claims that ensure low consumption growth volatility, i.e. that hedge against big swings in its consumption growth. To express the asset allocation decisions of the two countries in terms of more realistic assets (such as stocks and bonds), we would need to fully specify the assets that can be traded in the financial markets; the only requirement would be that the assets specified are sufficient for market completeness.[18]

3.3 International risk sharing

Consider the discounted marginal utility of domestic and foreign consumption, $\Lambda_t = e^{-\rho t}\frac{G_t}{C_t}$ and $\Lambda_t^* = e^{-\rho t}\frac{G_t^*}{C_t^*}$, respectively. The log pricing kernel (stochastic discount factor) of the domestic country is

$$d\log\Lambda_t = -\rho dt + d\log G_t - d\log C_t \tag{20}$$

and the log pricing kernel of the foreign country is

$$d\log\Lambda_t^* = -\rho dt + d\log G_t^* - d\log C_t^* \tag{21}$$

Note that $d\log\Lambda_t$ and $d\log\Lambda_t^*$ are the continuous-time equivalents of m_{t+1} and m_{t+1}^*, seen in the introduction.

It is shown in the Appendix (section A.1) that

$$d\log\Lambda_t^* - d\log\Lambda_t = d\log E_t \tag{22}$$

which is the continuous-time equivalent of (1) in logs.

As mentioned in the introduction, the international risk sharing puzzle is the coexistence of extremely high international pricing kernel correlation and relatively low international consumption growth rate correlation. In this model, the key for the explanation of the international risk sharing puzzle is the endogenously generated time-varying conditional consumption growth volatility discussed in the previous section. To understand how this endogeneity explains the puzzle, recall that: [19]

$$d\log G_t = drift - \delta\frac{G_t - l}{G_t}\left(\boldsymbol{\sigma}_t^{C\prime}\mathbf{dB}_t\right)$$

so the domestic log pricing kernel is:

$$d\log\Lambda_t = drift - \left(1 + \delta\frac{G_t - l}{G_t}\right)\left(\boldsymbol{\sigma}_t^{C\prime}\mathbf{dB}_t\right)$$

and, thus, the market price of domestic consumption risk is:

$$\boldsymbol{\eta}_t^C = \left(1 + \delta\frac{G_t - l}{G_t}\right)\boldsymbol{\sigma}_t^C$$

[18] Current work in progress examines portfolio choice in a two-good, two-country economy characterized by external habit formation and consumption home bias in preferences. It is shown that, under home bias, equilibrium portfolios are home biased; in fact, they are superbiased, in the Bennett and Young (1999) sense.

[19] This follows from an application of Itô's lemma to (6).

Similarly, the foreign log pricing kernel is:

$$d\log\Lambda_t^* = drift - \left(1+\delta\frac{G_t^*-l}{G_t^*}\right)\left(\boldsymbol{\sigma}_t^{C^*\prime}\mathbf{dB}_t\right)$$

so the market price of foreign consumption growth risk is:

$$\boldsymbol{\eta}_t^{C^*} = \left(1+\delta\frac{G_t^*-l}{G_t^*}\right)\boldsymbol{\sigma}_t^{C^*}$$

Note that the diffusion of the pricing kernel of each country - in other words, the market price of consumption risk - can be decomposed into two components: consumption growth volatility (the quantity of risk that the agent undertakes) and sensitivity to consumption growth shocks (which depends on conditional risk aversion).

To fix ideas, assume that the domestic country is conditionally more risk averse than the foreign country: $G_t > G_t^*$. First, consider the case of closed economies ($a = 1$, $a^* = 0$). In that case, as we have seen, each country consumes its endowment, so, under the assumption of homoskedastic endowment growth, conditional consumption growth volatility is constant for both countries (constant $\boldsymbol{\sigma}_t^C$ and $\boldsymbol{\sigma}_t^{C^*}$). Since each country's sensitivity to consumption growth shocks is increasing in its conditional risk aversion ($1+\delta\frac{G_t-l}{G_t}$ and $1+\delta\frac{G_t^*-l}{G_t^*}$ are increasing in G_t and G_t^*, respectively), the condition $G_t > G_t^*$ implies, ceteris paribus, that the domestic pricing kernel is conditionally more volatile than its foreign counterpart. This is the sensitivity effect. In that case, high correlation between $d\log G_t$ and $d\log G_t^*$ - and thus high correlation between the two log pricing kernels $d\log\Lambda_t$ and $d\log\Lambda_t^*$ - requires high correlation between the two countries' consumption growth processes. In other words, the market prices of consumption risk of the two countries are highly correlated only if their endowment growth processes are highly correlated.

However, this is not true for open economies: as we saw in the previous section, the condition $G_t > G_t^*$ also implies, ceteris paribus, that domestic conditional consumption growth volatility $\boldsymbol{\sigma}_t^C$ is lower than foreign conditional consumption growth volatility $\boldsymbol{\sigma}_t^{C^*}$. This is the consumption volatility effect and it has the opposite direction of the sensitivity effect, decreasing the *relative* conditional volatility of the domestic pricing kernel and increasing the *relative* volatility of the foreign kernel. Simply stated, for the domestic country, relatively high sensitivity to consumption growth risk is multiplied by relatively low consumption growth volatility. Exactly the opposite happens for the foreign country: relatively low sensitivity is multiplied by relatively high consumption growth volatility. The two components, sensitivity and volatility, have opposing effects on the *relative* market price of risk of the two countries. Apart from having opposite signs, the two components have similar magnitudes: this is because $\boldsymbol{\sigma}_t^C$ is roughly scaled by $\delta\frac{G_t^*-l}{G_t^*}$ (which is roughly foreign sensitivity), whereas $\boldsymbol{\sigma}_t^{C^*}$ is roughly scaled by $\delta\frac{G_t-l}{G_t}$ (roughly domestic sensitivity). The end result is that sensitivity and volatility balance each other out almost completely, bringing the two countries' market prices of consumption risk very close to each other and, thus, generating very high correlation between the two pricing kernels. The following corollary illustrates the above argument more formally.

Corollary 2 *Let $\boldsymbol{\phi}_t$ be the 2x1 vector such that*

$$\boldsymbol{\phi}_t'\mathbf{dB}_t = \frac{1}{D_t^c}\left(\left(ak_t^* + a^*k_t\right)\sigma^X dB_t^X + \left((1-a)k_t^* + (1-a^*)k_t\right)\sigma^Y dB_t^Y\right)$$

The domestic consumption growth rate process is:

$$d\log C_t = drift + \frac{1}{D_t^c}\left(a\sigma^X dB_t^X + (1-a)\sigma^Y dB_t^Y\right) + \delta\left(\frac{G_t^* - l}{G_t^*}\right)\boldsymbol{\phi}_t'\mathbf{dB}_t$$

and the foreign consumption growth rate process is:

$$d\log C_t^* = drift + \frac{1}{D_t^c}\left(a^*\sigma^X dB_t^X + (1-a^*)\sigma^Y dB_t^Y\right) + \delta\left(\frac{G_t - l}{G_t}\right)\boldsymbol{\phi}_t'\mathbf{dB}_t$$

Furthermore, the domestic log pricing kernel is:

$$\begin{aligned} d\log\Lambda_t &= drift - \left(1 + \delta\frac{G_t - l}{G_t}\right)\frac{1}{D_t^c}\left(a\sigma^X dB_t^X + (1-a)\sigma^Y dB_t^Y\right) \\ &\quad -\delta\left(\frac{G_t^* - l}{G_t^*}\right)\boldsymbol{\phi}_t'\mathbf{dB}_t - \delta^2\left(\frac{G_t - l}{G_t}\right)\left(\frac{G_t^* - l}{G_t^*}\right)\boldsymbol{\phi}_t'\mathbf{dB}_t \end{aligned}$$

and the foreign log pricing kernel is:

$$\begin{aligned} d\log\Lambda_t^* &= drift - \left(1 + \delta\frac{G_t^* - l}{G_t^*}\right)\frac{1}{D_t^c}\left(a^*\sigma^X dB_t^X + (1-a^*)\sigma^Y dB_t^Y\right) \\ &\quad -\delta\left(\frac{G_t - l}{G_t}\right)\boldsymbol{\phi}_t'\mathbf{dB}_t - \delta^2\left(\frac{G_t - l}{G_t}\right)\left(\frac{G_t^* - l}{G_t^*}\right)\boldsymbol{\phi}_t'\mathbf{dB}_t \end{aligned}$$

First, consider the two pricing kernels $d\log\Lambda_t$ and $d\log\Lambda_t^*$. Since the sensitivity parameter δ is large[20], the dominant term for both kernels is the last one, $-\delta^2\left(\frac{G_t-l}{G_t}\right)\left(\frac{G_t^*-l}{G_t^*}\right)\boldsymbol{\phi}_t' dB_t$, which is *identical* for both processes. This term represents the "canceling out" of the sensitivity and volatility effects described above. The fact that the dominant term is identical leads, of course, to unconditional correlation between the two kernels that is extremely close to 1. On the other hand, the two consumption growth processes do not include sensitivity terms, so there is nothing to counterbalance the volatility effect. Mathematically, the dominant term is $\delta\left(\frac{G_t^*-l}{G_t^*}\right)\boldsymbol{\phi}_t' dB_t$ for the domestic consumption growth rate and $\delta\left(\frac{G_t-l}{G_t}\right)\boldsymbol{\phi}_t' dB_t$ for the foreign consumption growth rate. It can be easily seen that those two expressions have values that are close to each other

[20] It is close to 80 in the calibration of Menzly et al. (2004).

when G_t^* and G_t are not too far apart. Thus, the correlation between those two terms is decreasing in the volatility of $\frac{G_t^*}{G_t}$: the more G_t^* and G_t diverge, the more the dominant terms of the two consumption growth rate processes diverge.

Finally, we can intuitively see how the assumption of external habit formation can help resolve the Backus and Smith (1993) puzzle. From (20), (21) and (22), we get:

$$d \log E_t = (d \log C_t - d \log C_t^*) + (d \log G_t^* - d \log G_t)$$

Real exchange rate changes are driven by both the consumption growth rate differential and an additional, habit-induced differential term, which breaks the perfect relationship between real exchange rates and consumption growth rates. In turn, this habit-related term depends, inter alia, on G_t and G_t^*, which are driven by past consumption realizations. In other words, it is not only present consumption that matters for real exchange rate changes; past consumption also matters.

3.4 International trade and the real exchange rate

We have so far described the equilibrium quantities of the model economy. The planner's problem can be easily decentralized to generate solutions for the terms of trade and, thus, the real exchange rate. The terms of trade are:

$$Q_t^* = \frac{1-a}{a} \frac{\omega_t}{\omega_t^*} \frac{\widetilde{X}_t}{\widetilde{Y}_t} \tag{23}$$

The relative price of the foreign good Q_t^* depends on two ratios: the endowment ratio $\frac{\widetilde{X}_t}{\widetilde{Y}_t}$ and the risk aversion ratio $\frac{G_t^*}{G_t}$, the latter through the share ratio $\frac{\omega_t}{\omega_t^*}$. The dependence of the terms of trade on the endowment ratio is not surprising, as it is well established in standard two-country models: the endowment ratio reflects the relative scarcity of the two goods; high $\widetilde{X}_t$ relative to $\widetilde{Y}_t$ means that the foreign good is relatively scarcer and thus commands a high relative price Q_t^*. What is new in this model is the dependence on the risk aversion ratio $\frac{G_t^*}{G_t}$. Recall that under home bias ($a > a^*$), $\frac{\omega_t}{\omega_t^*}$ is increasing in $\frac{G_t^*}{G_t}$, so the terms of trade are increasing in the ratio of risk aversions. This is because, as seen earlier, high values of $\frac{G_t^*}{G_t}$ correspond to elevated consumption demand in the foreign country and reduced consumption demand in the domestic country. If consumption in both countries is home biased, then most of the high foreign consumption demand is expressed as high demand for the foreign good; correspondingly, there is low demand for the domestic good. The end result is a high relative price Q_t^* for the foreign good.

It is also important to note that the assumption of external habit preferences, by adding dependence on the risk aversion ratio $\frac{G_t^*}{G_t}$, significantly increases the volatility of the terms of trade: Q_t^* is now driven by the two endowment shocks through *two* mechanisms: there is a direct effect of the shocks though the endowment ratio $\frac{\widetilde{X}_t}{\widetilde{Y}_t}$ and an indirect (and larger) effect through the risk aversion ratio $\frac{G_t^*}{G_t}$. Those effects reinforce each other: a relative positive domestic endowment shock tends to increase both $\frac{\widetilde{X}_t}{\widetilde{Y}_t}$ and $\frac{G_t^*}{G_t}$, thus greatly enhancing terms of trade volatility vis-a-vis the benchmark of standard preferences.

Since the real exchange rate is proportional (in logs) to the terms of trade, the two variables share the same characteristics. Specifically, the real exchange rate is:

$$E_t = \left(\frac{a}{a^*}\right)^{a^*} \left(\frac{1-a}{1-a^*}\right)^{1-a^*} \left(\frac{\omega_t}{\omega_t^*}\right)^{a-a^*} \left(\frac{\widetilde{X}_t}{\widetilde{Y}_t}\right)^{a-a^*} \tag{24}$$

Unsurprisingly, E_t is increasing in the endowment ratio: a positive endowment shock in a country depreciates its currency in real terms. Furthermore, under home bias, an increase in the risk aversion ratio $\frac{G_t^*}{G_t}$ leads to a real depreciation of the domestic currency; this is because an increase in Q_t^* increases the foreign price level P_t^* much more than it increases the domestic price level P_t. What is true for the volatility of the terms of trade is also true for the volatility of the real exchange rate: the addition of external habit preferences greatly enhances real exchange rate volatility.

Finally, the domestic net exports ratio, i.e. the ratio of the value of net exports over the value of the endowment, is:

$$NX_t = \frac{\widetilde{X}_t - C_t P_t}{\widetilde{X}_t} = \frac{X_t^* - Y_t Q_t^*}{\widetilde{X}_t} = 1 - \frac{\omega_t}{a} \tag{25}$$

It is important to note that the net exports ratio only depends on the risk aversion ratio $\frac{G_t^*}{G_t}$; only relative risk aversion matters for the external sector. Furthermore, the domestic net exports ratio is increasing in $\frac{G_t^*}{G_t}$: high values of the risk aversion ratio mean that, as we have seen before, goods flow from the domestic to the foreign country, since the latter needs consumption more.

3.5 Wealth and welfare weights

To close the model, we need to calculate the endogenous welfare weights λ and λ^*. The following proposition, proven in the Appendix, illustrates the connections between wealth and welfare weights.

Proposition 3 *Domestic wealth W_t in units of the domestic good is:*

$$W_t = \frac{\rho G_t + k\bar{G}}{a\lambda G_t + a^*\lambda^* G_t^*} \frac{\lambda \widetilde{X}_t}{\rho(\rho + k)} \tag{26}$$

and foreign wealth W_t in units of the foreign good is:

$$W_t^* = \frac{\rho G_t^* + k\bar{G}}{(1-a)\,\lambda G_t + (1-a^*)\,\lambda^* G_t^*} \frac{\lambda^* \widetilde{Y}_t}{\rho\,(\rho+k)} \tag{27}$$

where $\lambda = \frac{a^*\left(\rho G_0^* + k\bar{G}\right)}{(1-a)\left(\rho G_0 + k\bar{G}\right) + a^*\left(\rho G_0^* + k\bar{G}\right)}$ *and* $\lambda^* = 1 - \lambda = \frac{(1-a)\left(\rho G_0 + k\bar{G}\right)}{(1-a)\left(\rho G_0 + k\bar{G}\right) + a^*\left(\rho G_0^* + k\bar{G}\right)}$. *Initial domestic wealth, as a proportion of global wealth, is:*

$$\frac{W_0}{W_0 + W_0^* Q_0^*} = \frac{a^*}{1 - a + a^*} \tag{28}$$

Each country's wealth, in units of its own good, is increasing in its endowment and decreasing in the other country's risk aversion. However, the sign of dependence on its own risk aversion is not clear, as it depends on the parameter values and the value of the other country's risk aversion. Nevertheless, it is easy to characterize the wealth ratio of the two countries; it is:

$$\frac{W_t}{W_t^* Q_t^*} = \frac{\lambda}{\lambda^*} \frac{\rho G_t + k\bar{G}}{\rho G_t^* + k\bar{G}}$$

The wealth ratio is increasing in domestic risk aversion G_t and decreasing in foreign risk aversion G_t^*.

The domestic share of global initial wealth $\frac{W_0}{W_0 + W_0^* Q_0^*}$ is increasing in both a and a^*. This makes sense: the stronger the preference for the domestic good from either the domestic (a) or the foreign (a^*) country, the wealthier the domestic country is. In the limit, as the domestic country becomes completely home biased ($a \to 1$) but the foreign country is not ($a^* \in (0,1)$), then the domestic country has all the wealth ($\frac{W_0}{W_0 + W_0^* Q_0^*} \to 1$); conversely, when the foreign country is completely home biased ($a^* \to 0$) and the domestic country is not ($a \in (0,1)$), then the foreign country has all the wealth ($\frac{W_0}{W_0 + W_0^* Q_0^*} \to 0$). This is intuitive: for example, when the domestic country is completely home biased and the foreign country is not, the foreign country wants to import from the domestic country, but it has nothing that the domestic country wants; the terms of trade Q^* approach zero, so the foreign country has a valueless endowment. It can be shown that when both countries are completely home biased ($a = 1$ and $a^* = 0$), the initial wealth ratio is indeterminate: since no country has preferences over both goods, there is no way to determine their relative price Q^* and, ultimately, the relative wealth of the two countries.

The domestic welfare weight λ is increasing in both a and a^*. Furthermore, λ is decreasing in G_0 and increasing in G_0^*: the more initially risk averse country has a lower welfare weight, ceteris paribus. The limit behavior is illuminating:

$$\lim_{G_0 \to \infty} \lambda = 0 \text{ and } \lim_{G_0^* \to \infty} \lambda = 1$$

Furthermore, λ has the same behavior as $\frac{W_0}{W_0+W_0^* Q_0^*}$ for boundary values of the two parameters: it approaches 1 (0) when the domestic (foreign) country approaches complete home bias and it is indeterminate when both countries are completely home biased.

For $G_0 = G_0^*$:

$$\lambda = \frac{a^*}{1 + a^* - a} = \frac{W_0}{W_0 + W_0^* Q_0^*}$$

Thus, if initial risk aversion is equal for both countries, λ is equal to the proportion of initial wealth that the domestic country owns, so λ has a very natural interpretation. However, this is not true for $G_0 \neq G_0^*$.

4 Asset prices

So far I have assumed complete markets, without explicitly specifying the securities in which the agents can invest. Under market completeness, all assets can be priced by no arbitrage, using the prices of Arrow-Debreu securities. In this section, I consider four assets: two *total wealth portfolios*, the domestic and the foreign one, and two locally riskless assets, the *domestic bond* and the *foreign bond*. The domestic (foreign) total wealth portfolio is the asset that pays as dividend, each period, the endowment of the domestic (foreign) country. The net supply of each of those two portfolios is normalized to one. The domestic bond is a locally riskless asset in terms of the domestic good, in the sense that its return *in terms of the domestic good* is the same across states of the world; similarly for the foreign bond. Both bonds are in zero net supply. The price of the four assets is, respectively, V_t, V_t^*, D_t and D_t^*; all prices are denoted in units of the local good, so V_t and D_t are expressed in units of the domestic good and V_t^* and D_t^* are expressed in units of the foreign good.

4.1 Risk-free rates

The price of the domestic bond satisfies the stochastic differential equation $dD_t = r_t^f D_t dt$, where r_t^f is the continuously compounded domestic risk-free rate, i.e. the real rate of return demanded from a riskless investment in the domestic good. Similarly, the price of the foreign bond solves $dD_t^* = r_t^{f*} D_t^* dt$. Note that neither of those bonds is riskless in terms of any of the two *consumption* baskets. Thus, there are consumption risk premia associated with both of those bonds; those premia are, in effect, compensation for terms of trade risk.[21]

Proposition 4, the proof of which can be found in the Appendix, reports the domestic and foreign risk-free rates.

Proposition 4 *Let* $\mathbf{e}_1 \equiv [1\ 0]'$, $\mathbf{e}_2 \equiv [0\ 1]'$. *Also, denote* $\boldsymbol{\sigma}_t^G = \delta \left(\frac{G_t - l}{G_t} \right) \boldsymbol{\sigma}_t^C$ *and* $\boldsymbol{\sigma}_t^{G^*} =$

[21] We can also define *consumption bonds*, with the domestic (foreign) consumption bond being locally riskless in terms of the domestic (foreign) consumption basket. See the Appendix (section A.2) for a discussion.

$\delta\left(\frac{G_t^*-l}{G_t^*}\right)\boldsymbol{\sigma}_t^{C^*}$. *The domestic risk-free rate is:*

$$\begin{aligned} r_t^f &= \rho+\mu_t^X+k\left[\omega_t\left(\frac{G_t-\bar{G}}{G_t}\right)+(1-\omega_t)\left(\frac{G_t^*-\bar{G}}{G_t^*}\right)\right] \\ &\quad -\left[\omega_t\boldsymbol{\sigma}_t^G+(1-\omega_t)\boldsymbol{\sigma}_t^{G^*}\right]'\boldsymbol{\Sigma}\mathbf{e}_1\sigma^X-\frac{1}{2}\left(\sigma^X\right)^2 \end{aligned} \tag{29}$$

and the foreign risk-free rate is:

$$\begin{aligned} r_t^{f*} &= \rho+\mu_t^Y+k\left[\omega_t^*\left(\frac{G_t-\bar{G}}{G_t}\right)+(1-\omega_t^*)\left(\frac{G_t^*-\bar{G}}{G_t^*}\right)\right] \\ &\quad -\left[\omega_t^*\boldsymbol{\sigma}_t^G+(1-\omega_t^*)\boldsymbol{\sigma}_t^{G^*}\right]'\boldsymbol{\Sigma}\mathbf{e}_2\sigma^Y-\frac{1}{2}\left(\sigma^Y\right)^2 \end{aligned} \tag{30}$$

I focus on the domestic risk-free rate r_t^f; for r_t^{f*}, the analysis is identical. Unsurprisingly, the first term is the subjective discount rate ρ: the higher the agents discount the future, the higher the interest rate has to be. The next two terms are marginal utility-smoothing terms: μ_t^X is the familiar endowment-smoothing term, while the second term results from the agents' desire to smooth their conditional risk aversion. Specifically, when G_t and G_t^* are above their unconditional mean $\bar{G}$, marginal utility is high, so the agents' willingness to save is low and equilibrium interest rates are high. Importantly, *both* risk aversions matter for *both* risk-free rates: the risk aversion-smoothing term depends on a weighted average of the two percentage deviations from unconditional risk aversion, with the weighted average largely depending on the home bias parameters a and a^*. The last two terms are related to precautionary savings, so they enter with a negative sign: the more conditionally volatile domestic or foreign consumption growth (or, less importantly, domestic endowment growth) are, the more precautionary savings will the domestic agent desire, decreasing the equilibrium riskless rate.

4.2 Total wealth portfolio prices

After considering the two bonds, we now turn to the two total wealth portfolios. Proposition 5, proven in the Appendix, presents the equilibrium price of the two portfolios.

Proposition 5 *The price-dividend ratio of the domestic total wealth portfolio is:*

$$\frac{V_t}{\widetilde{X}_t}=\frac{1}{\rho}\left(\frac{k}{\rho+k}\frac{(a\lambda+a^*\lambda^*)\bar{G}}{a\lambda G_t+a^*\lambda^*G_t^*}+\frac{\rho}{\rho+k}\right) \tag{31}$$

and the price-dividend ratio of the foreign total wealth portfolio is:

$$\frac{V_t^*}{\widetilde{Y}_t}=\frac{1}{\rho}\left(\frac{k}{\rho+k}\frac{((1-a)\lambda+(1-a^*)\lambda^*)\,\bar{G}}{(1-a)\lambda G_t+(1-a^*)\lambda^*G_t^*}+\frac{\rho}{\rho+k}\right) \tag{32}$$

To realize the effects of international trade on asset prices, first consider the closed economy case ($a = 1$ and $a^* = 0$). Then, it can easily be shown that the price-dividend ratio of the domestic total wealth portfolio is

$$V_t = \frac{1}{\rho}\left(\frac{k}{\rho+k}\frac{\bar{G}}{G_t} + \frac{\rho}{\rho+k}\right)\widetilde{X}_t$$

which, unsurprisingly, is the solution obtained in Menzly et al. (2004). In the power utility benchmark, which here obtains by setting $G_t = \bar{G}$, the price-dividend ratio is $\frac{1}{\rho}$, so changes in the endowment have a linear impact on prices. With habit preferences, however, the price-dividend ratio is no longer constant, but varies with G_t. In this case, a positive (for example) shock to $\widetilde{X}_t$ increases V_t both directly and indirectly, the latter through its negative effect on G_t. Thus, habits considerably magnify the effects of endowment shocks on asset prices by adding a second, multiplicative effect of endowment shocks on asset prices.

In an open economy, this dual effect of endowment shocks on asset prices is the same, with the difference being that what matters is not just the local endowment shock, but also the foreign one. The two endowment shocks affect prices in a more complicated way than in the closed economy case. Specifically, the domestic endowment shock affects both G_t and G_t^*, since both processes are driven by consumption growth shocks, and both consumption growth shocks are, in turn, affected by both endowment shocks (consider the solution for the two countries' consumption growth rate in the previous section). Under home biased preferences, the two shocks are not equally important, of course: the domestic endowment shock affects G_t more than G_t^*. Similarly, the foreign endowment shock affects both G_t and G_t^*, but primarily the latter. Furthermore, only the domestic shock has a direct effect on V_t, through $\widetilde{X}_t$, and, conversely, only the foreign shock directly affects V_t^*.

The expressions for the two portfolios' price-dividend ratios, (31) and (32), are economically intuitive. Since each country's total wealth portfolio represents claims to its endowment good, the price of the two total wealth portfolios will depend on both domestic and foreign demand for the endowment goods. As shown in the previous section, under certain conditions, the domestic welfare weight λ is equal to the proportion of global wealth that the domestic country initially owns. Thus, the domestic price-dividend ratio will depend on both countries' time-varying risk aversions, weighted by each country's wealth (λ, λ^*) and desire for the domestic good (a, a^*). As a corollary, the foreign country's time-varying risk aversion will have a big impact on the domestic country's price-dividend ratio if either the foreign country is wealthy compared to the domestic country (i.e. λ^* is high relative to λ), or if it has a strong preference for the domestic good (i.e. a^* is high).This means, for example, that US risk aversion has a large effect on other countries' price-dividend ratios (and hence asset prices and returns), since the US is large compared to almost all other economies. Conversely, foreign countries' risk aversions have a relatively small effect on US asset prices, since the US economy is both large and relatively closed. Furthermore, it is the asset prices of small countries and countries with a significant volume of exports (large a^*) that will be heavily affected by foreign risk preferences G_t^*.[22]

4.3 Total wealth portfolio excess returns

After analyzing prices, we need to examine excess returns $\frac{dV_t}{V_t} + \frac{\widetilde{X}_t}{V_t} dt - r_t^f dt$ and $\frac{dV_t^*}{V_t^*} + \frac{\widetilde{Y}_t}{V_t^*} dt - r_t^{f*} dt$ for the domestic and foreign total wealth portfolio, respectively. The domestic total wealth portfolio pays domestic good dividends $\left\{ \widetilde{X}_t \right\}_{t=0}^{\infty}$, discounted by the domestic good marginal utility Ξ_t,[23] which satisfies

$$\frac{d\Xi_t}{\Xi_t} = -r_t^f dt - \boldsymbol{\eta}_t' \mathbf{dB}_t$$

where $\boldsymbol{\eta}_t$ is the market price of domestic good risk. Similarly for the foreign total wealth portfolio and the foreign good marginal utility Ξ_t^*. An explicit solution for the excess returns of the two portfolios is provided in the following proposition, the proof of which can be found in the Appendix.[24]

Proposition 6 *Let* $\mathbf{e}_1 \equiv [1\ 0]'$, $\mathbf{e}_2 \equiv [0\ 1]'$ *. Also, denote* $\boldsymbol{\sigma}_t^G = \delta \left(\frac{G_t - l}{G_t} \right) \boldsymbol{\sigma}_t^C$ *and* $\boldsymbol{\sigma}_t^{G^*} = \delta \left(\frac{G_t^* - l}{G_t^*} \right) \boldsymbol{\sigma}_t^{C^*}$*. The excess return, in terms of the domestic good, of the domestic total wealth portfolio is*

$$dR_t^e = \left(\boldsymbol{\eta}_t' \boldsymbol{\Sigma} \boldsymbol{\sigma}_t^R \right) dt + \boldsymbol{\sigma}_t^{R\prime} \mathbf{dB}_t$$

where $\boldsymbol{\eta}_t$ *is the market price of domestic good risk, given by*

$$\boldsymbol{\eta}_t = \sigma^X \mathbf{e}_1 + \left(\omega_t \boldsymbol{\sigma}_t^G + (1 - \omega_t) \boldsymbol{\sigma}_t^{G^*} \right) \tag{33}$$

and $\boldsymbol{\sigma}_t^R$ *is the diffusion process of the domestic total wealth portfolio excess return, given by*

$$\boldsymbol{\sigma}_t^R = \sigma^X \mathbf{e}_1 + \frac{(a\lambda + a^*\lambda^*) k \bar{G}}{(a\lambda + a^*\lambda^*) k \bar{G} + \rho \left(a\lambda G_t + a^*\lambda^* G_t^* \right)} \left(\omega_t \boldsymbol{\sigma}_t^G + (1 - \omega_t) \boldsymbol{\sigma}_t^{G^*} \right) \tag{34}$$

[22] Note that country wealth refers to aggregate wealth, not per capita wealth. In this model, each country's population has been normalized to 1, so per capita and aggregate figures coincide. However, it should be stressed that, if the model is to be mapped to real data, all quantities mentioned are aggregate quantities. It can be shown that the results for aggregate variables are identical under any assumption regarding the two countries' population measures, as long as both measures are constant over time. The following section discusses the mapping of the model to empirical data.

[23] In fact, the two agents, domestic and foreign, have different, but proportional, marginal utility of the domestic good. Ξ_t is defined such that $\Xi_t = \lambda MU_t^X = \lambda^* MU_t^{X^*}$; thus, Ξ_t is proportional to both the domestic (MU_t^X) and the foreign ($MU_t^{X^*}$) marginal utility of the domestic good. For more details, see the Appendix.

[24] There is a tight connection between the marginal utility of domestic and foreign consumption Λ_t and Λ_t^*, respectively, and the marginal utility of the domestic and foreign good Ξ_t and Ξ_t^*, respectively. Details are provided in the Appendix (section A.2).

The excess return, in terms of the foreign good, of the foreign total wealth portfolio is

$$dR_t^{e*} = \left(\boldsymbol{\eta}_t^{*\prime}\boldsymbol{\Sigma}\boldsymbol{\sigma}_t^{R^*}\right)dt + \boldsymbol{\sigma}_t^{R^*\prime}\mathbf{dB}_t$$

where $\boldsymbol{\eta}_t^*$ *is the market price of foreign good risk, given by*

$$\boldsymbol{\eta}_t^* = \sigma^Y\mathbf{e}_2 + \left(\omega_t^*\boldsymbol{\sigma}_t^G + (1-\omega_t^*)\boldsymbol{\sigma}_t^{G^*}\right) \tag{35}$$

and $\boldsymbol{\sigma}_t^{R^*}$ *is the diffusion process of the foreign total wealth portfolio excess return, given by*

$$\begin{aligned}\boldsymbol{\sigma}_t^{R^*} &= \sigma^Y\mathbf{e}_2 + \frac{((1-a)\,\lambda + (1-a^*)\,\lambda^*)k\bar{G}}{((1-a)\,\lambda + (1-a^*)\,\lambda^*)k\bar{G} + \rho\left((1-a)\,\lambda G_t + (1-a^*)\,\lambda^* G_t^*\right)} \times \\ &\quad \left(\omega_t^*\boldsymbol{\sigma}_t^G + (1-\omega_t^*)\boldsymbol{\sigma}_t^{G^*}\right)\end{aligned} \tag{36}$$

For each portfolio, the expected excess return *in terms of the local good* is determined by the covariation of the portfolio return with the relevant marginal utility growth: since the domestic (foreign) portfolio pays domestic (foreign) good dividends, its risk premium is compensation for domestic (foreign) good risk. As in the closed economy benchmark, holding the domestic total wealth portfolio is risky in terms of the domestic good, as it tends to generate low payoffs exactly when domestic good marginal utility is high, and vice versa. As it can be easily seen from the functional forms of $\boldsymbol{\sigma}_t^R$ and $\boldsymbol{\eta}_t$, it will have a positive risk premium. The same applies to the foreign total wealth portfolio: it pays a lot in terms of the foreign asset exactly when the foreign asset is not very valuable in marginal utility terms. After establishing that the two portfolios have positive risk premia, we can now examine the magnitude of those premia. Since the sensitivity parameter δ is very high, the habit-induced second term of $\boldsymbol{\eta}_t$ (and $\boldsymbol{\eta}_t^*$) contributes to a big increase of market price of risk over the power utility benchmark. In other words, the habit-induced multiplicative mechanism that generates high risk premia in models of closed economies retains its potency in open economies.

Furthermore, the market price of risk is time-varying: both $\boldsymbol{\eta}_t$ and $\boldsymbol{\eta}_t^*$ are increasing in both domestic and foreign conditional consumption growth volatilities $\boldsymbol{\sigma}_t^C$ and $\boldsymbol{\sigma}_t^{C^*}$. In addition to that, returns are conditionally heteroskedastic, as both $\boldsymbol{\sigma}_t^R$ and $\boldsymbol{\sigma}_t^{R^*}$ are time-varying. As a result, risk premia of the two total wealth portfolios ($\boldsymbol{\eta}_t^{\prime}\boldsymbol{\Sigma}\boldsymbol{\sigma}_t^R$ for the domestic one and $\boldsymbol{\eta}_t^{*\prime}\boldsymbol{\Sigma}\boldsymbol{\sigma}_t^{R^*}$ for the foreign one) are also time-varying, as in the closed economy case.

4.4 Asset prices and exchange rates

As seen in (24), the addition of habits generates additional variability in the real exchange rate through the mechanism of time-varying risk aversions G and G^*. Since time-varying risk aversion also generates time variation in price-dividend ratios, we can clearly see the relationship between asset prices and the real exchange rate by rewriting (24) as follows:

$$E_t = \frac{a^a(1-a)^{1-a}}{(a^*)^{a^*}(1-a^*)^{1-a^*}} \left(\frac{(1-a)\lambda + (1-a^*)\lambda^*}{a\lambda + a^*\lambda^*}\right)^{a-a^*} \left(\frac{\frac{V_t}{\widetilde{X}_t} - \frac{1}{\rho+k}}{\frac{V_t^*}{\widetilde{Y}_t} - \frac{1}{\rho+k}}\right)^{a-a^*} \left(\frac{\widetilde{X}_t}{\widetilde{Y}_t}\right)^{a-a^*}$$

In our model, real exchange rate volatility is caused by two economic mechanisms: time variation in relative endowments (endowment mechanism) and time variation in price-dividend ratios (asset pricing mechanism). Since the variability of price-dividend ratios is typically much higher than the variability of macroeconomic variables, the most important mechanism for real exchange rate volatility is the asset pricing mechanism. In the absence of habits, $\frac{V_t}{\widetilde{X}_t}$ and $\frac{V_t^*}{\widetilde{Y}_t}$ are constant; the asset pricing mechanism shuts down, leaving only the endowment mechanism to generate real exchange rate volatility. This is the reason standard international macroeconomic models, which do not generate time-varying price-dividend ratios, severely undershoot the empirical level of real exchange rate volatility.

5 Simulation

5.1 Definitions and data

In order to calibrate the model, I discretize it at the quarterly frequency; the United States is the domestic country and the United Kingdom is the foreign country. Since this paper considers an endowment model which includes neither investment nor government spending, real endowment is mapped to the sum of consumption of non-durables and services (NDS) and total net exports, in real per capita terms.[25] This is consistent with Verdelhan (2008b) and Colacito and Croce (2008b). It should be noted that all imports and exports (regardless of country of origin and destination, respectively) are taken into account for the calculation of each country's endowment. On the other hand, exports and imports in the model are mapped to bilateral US-UK trade flows. The adopted mapping of model variables to real-world variables is meant to accommodate the two-country nature of the model under consideration. Specifically, regarding endowments, consumption data already include imported goods and services from all the other countries of the world, so, in order to derive the correct measure of home production, it is imperative that total imports are subtracted from consumption (and total exports added

[25] As previously mentioned, the population of each country is normalized to 1, but this normalization does not affect the results of the model; the analysis is identical under any assumption regarding the two countries' population measures. However, the model cannot accommodate population growth; population measures have to be constant. To correct for population growth in the empirical data, we can either adjust aggregate data for population growth, or use per capita data. The only difference between the two approaches regards the scale of the model, i.e. the level of quantities; growth rates, scaled quantities (such as price-dividend ratios and net exports ratios) and asset returns are unaffected. In this calibration, I follow the second approach and use per capita data.

back). On the other hand, if model trade flows were mapped to total trade flows, trade between the US and the UK would be greatly exaggerated: the calibrated model would be pushed to generate unrealistic trade patterns between the two countries. [26]

The sample period is 1975:Q1 to 2007:Q2, for a total of 130 quarterly observations. The data on seasonally-adjusted consumption, total imports and total exports, nominal and real, are from the US Bureau of Economic Analysis (BEA) and the UK Office for National Statistics (ONS). Implicit price deflators are constructed as the ratio of nominal to real quantities. Population data, used to calculate per capita figures where necessary, are constructed as the ratio of nominal GDP over nominal GDP per capita. Non-seasonally adjusted bilateral US-UK trade data (in USD) are from the BEA; they are seasonally adjusted with the US Census Bureau's X12 seasonal adjustment program and, where necessary, are converted to GBP using the quarterly average exchange rate from the IMF International Financial Statistics (IFS). The total wealth portfolio for each country is proxied by the corresponding country's Datastream equity index; real returns are constructed using the Total Return Index, while the series for the price-dividend ratio is constructed using the Total Return Index and the Price Index series.[27] The nominal US risk-free rate is proxied by the 3-month Treasury bill rate (from CRSP) and the UK risk-free rate is proxied by the UK government 3-month bill yield (from ONS). The data for the nominal end-of-quarter USD/GBP exchange rate are from MSCI. CPI data are from the IMF IFS. Nominal endowment and nominal NDS consumption are deflated by the corresponding country's CPI. The terms of trade Q^* are the ratio of the implicit price deflator for total UK exports over the implicit price deflator for total US exports.

5.2 The endowment processes

In the previous sections, the drift processes of the endowments were left unspecified. To calibrate the model, I have to assume specific functional forms for the two endowment drifts. I choose the functional form in a way that imposes on the drifts some structure based on empirical evidence. Specifically, I would like to allow for the possibility that the real exchange rate between the two countries is stationary.[28] From (24), it follows that real exchange rate stationarity requires that the ratio z of the two endowment processes be stationary. This is intuitive: a non-stationary ratio of the two endowments would imply that the output of one economy would almost surely approach zero as a proportion of the other country's endowment as $t \to \infty$, as illustrated by Cochrane, Longstaff and Santa-Clara (2007).

[26] Under the adopted mapping, the part of the US (UK) endowment that is, in reality, exported to countries other than the UK (US) is implicitly assumed to be consumed by the US (UK) representative agent.

[27] Specifically, the Total Return Index $\tilde{P}$, the Price Index P and the dividend D are connected by the following relationship: $\frac{\tilde{P}_{t+1}}{\tilde{P}_t} = \frac{P_{t+1}+D_{t+1}}{P_t}$. Then, it can easily be seen that $\frac{P_{t+1}}{D_{t+1}} = \frac{P_{t+1}}{P_t}\left(\frac{\tilde{P}_{t+1}}{\tilde{P}_t} - \frac{P_{t+1}}{P_t}\right)^{-1}$. Bansal, Dittmar and Lundblad (2005) use an equivalent procedure to calculate portfolio dividends.

[28] For a discussion of the empirical evidence regarding the stationarity of real exchange rates, see Sarno and Taylor (2002).

For this purpose, I assume that the log of the ratio of the two endowment processes $z_t \equiv \frac{\widetilde{Y}_t}{\widetilde{X}_t}$ is an Ornstein-Uhlenbeck (first order autoregressive) process:

$$d \log z_t = \theta(\log \bar{z} - \log z_t)dt + \sigma^z dB_t^z \tag{37}$$

where $\sigma^z = \left(\sigma^X\right)^2 + \left(\sigma^Y\right)^2 - 2\sigma^X\sigma^Y\rho^{XY}$ and $dB_t^z \equiv \frac{\sigma^Y dB_t^Y - \sigma^X dB_t^X}{\sigma^z}$. The endowment ratio z_t is positive (almost surely) and mean reverts to its long-run level $\bar{z}$. The speed of mean reversion (in logs) is θ and the volatility of the (log) ratio is σ^z.

The process for $\widetilde{X}_t$ is assumed to be:

$$d \log \widetilde{X}_t = \left[\mu - \psi\theta(\log \bar{z} - \log z_t)\right] dt + \sigma^X dB_t^X \tag{38}$$

Parameter μ is the unconditional domestic endowment growth rate. Parameter ψ measures the degree of adjustment done by $\widetilde{X}_t$. Specifically, since $\log z_t$ is stationary, $\log \widetilde{X}_t$ and $\log \widetilde{Y}_t$ are cointegrated, so their paths are not independent of each other: either $\log \widetilde{X}_t$ adjusts to $\log \widetilde{Y}_t$ (in other words, $\log \widetilde{X}_t$ error-corrects), or $\log \widetilde{Y}_t$ error-corrects or both do. For $\psi = 0$, $\log \widetilde{X}_t$ does not adjust at all to the movements of $\log \widetilde{Y}_t$, so stationarity is preserved solely by the adjustment of $\log \widetilde{Y}_t$. On the other hand, for $\psi = 1$, all adjustment is done by $\log \widetilde{X}_t$. For $\psi \in (0, 1)$, both processes error-correct.

The process for the foreign country's endowment $\widetilde{Y}_t = z_t \widetilde{X}_t$ is given by an application of Itô's lemma using (38) and (37):

$$d \log \widetilde{Y}_t = \left[\mu + (1 - \psi)\theta(\log \bar{z} - \log z_t)\right] dt + \sigma^Y dB_t^Y \tag{39}$$

The long-run mean of the foreign endowment growth rate is also μ, since, to achieve cointegration, the two countries must grow at the same rate, on average. The proportion of global error-correction performed by the foreign economy is $1 - \psi$.

It should be noted that the non-cointegration case is a special case in this setup, achieved when $\theta = 0$, in which case both log endowment processes $\log \widetilde{X}$ and $\log \widetilde{Y}$ are geometric Brownian motions and the log endowment ratio $\log z$ is a scaled Brownian motion.

5.3 Parameter calibration

The model includes 14 parameters in total, 7 endowment-related and the rest preference-related. The 7 endowment-related parameters (μ, θ, ψ, $\bar{z}$, σ^X, σ^Y and ρ^{XY}) are calibrated using US and UK endowment data (as defined in the previous section). Specifically, discretizing (37) and (38), I get 7 moment conditions, which are used to infer the 7 endowment parameters using exactly identified GMM estimation; details are provided in the Appendix (section A.3). The parameter estimates, along with their standard errors, appear in Table 1. The point estimate of

θ, the mean-reversion speed of $\log z$ is 0.05 (0.19 annualized) and it appears that cointegration is achieved because the UK endowment adjusts to the US one, rather than vice versa: the point estimate for ψ is less than 0.02 and not statistically significant, which implies that more than 98% (if not all) of the endowment adjustment is done by the UK. Clearly, the US economy has an impact on the UK one, but not vice versa. Regarding $\bar{z}$, it should be noted that its value only affects the scale of the model variables; it can be normalized to 1 without any effect to the moments. Lastly, it appears that the US economy is less volatile than the UK one: the point estimate of US quarterly endowment growth rate standard deviation σ^X is 0.74% (1.49% annualized), while the corresponding UK volatility σ^Y is more than double that, standing at 1.95% (3.91% annualized). The correlation of the two endowment growth rates is around 0.16 and is marginally statistically significant, with its t-statistic being 1.98.

Regarding the 5 habit-related preferences parameters, the values for ρ, δ, $\bar{G}$ and l are the ones used in Menzly et al. (2004), while the value of k, the speed of mean reversion of G, is adjusted downwards (from 0.16 in Menzly et al. (2004) to 0.12) to get a better of with the return data.[29] Regarding the home bias parameters a and a^*, they are calibrated to match the share of the domestic good expenditure and foreign good expenditure in each country's consumption expenditure. In the model, the share of foreign good expenditure in the domestic consumption expenditure is

$$\frac{Y_t Q_t^*}{C_t P_t} = 1 - a$$

while the share of domestic good expenditure in the foreign consumption expenditure is

$$\frac{X_t^*}{C_t^* P_t^*} = a^*$$

Since, on average, imports from the UK represent 1.0% of US consumption expenditure and US imports correspond to 8.2% of UK consumption, the calibrated values are $a = 0.990$ and $a^* = 0.082$. For those values of the home bias parameters, the US welfare weight is $\lambda = 0.89$, with the UK weight being $\lambda^* = 0.11$; thus, the US has 8.2 times the weight of the UK.

The values for the calibrated parameters are presented in Table 2.

5.4 Simulation results

I simulate 10,000 sample paths of the model economy, each consisting of 170 quarterly observations. The system is initialized at the steady state ($z_1 = \bar{z}$, $G_1 = G_1^* = \bar{G}$) and I adopt the normalization $\widetilde{X}_1 = 1$. Of the 170 observations, the first 40 (10 years) are discarded to

[29] In Menzly et al. (2004) the 5 habit-related preference parameters are calibrated to match the following 5 moment conditions for US data: $E(P/D)$, $E(R^e)$, $var(R^e)$, $E(r^f)$ and $var(r^f)$. Since the Menzly et al. calibration is designed to match asset pricing moments, the downward adjustment of k in this paper, aimed at a better fit of the asset pricing moments, is consistent with the spirit of the original calibration.

reduce the dependence on initial conditions, so, at the end, each sample path consists of 130 observations, as many as available in the dataset. All moments of interest are calculated for each of the 10,000 simulated paths. For each of the moments of interest, Table 3 presents the sample average across the 10,000 simulations, as well as the 2.5% and 97.5% percentiles across simulations.

5.4.1 Endowment, consumption and risk sharing

Panel A presents moments pertaining to endowment and consumption growth rates and risk sharing. To begin with, the simulated model adequately captures the relative size of the two economies: in the model, the US economy is, on average, 8.23 times the size of the UK economy, very close to the empirical value of 8.29. As outlined above, the mean, standard deviation and correlation of endowment growth rates are calibrated moments. However, the model also adequately matches the autocorrelation of the two countries' endowment growth rates, which hints that the postulated functional form for the endowment processes is not implausible.

Since consumption is endogenous in the model, the most important moments of Panel A are the ones related to consumption growth rates. Given the endogeneity of consumption, the performance of the model is unambiguously good. Simulated consumption growth rate means and, more importantly, standard deviations are very close to their empirical counterparts. Although consumption growth rate autocorrelation is not perfectly matched, the disparities between simulated and empirical data are not big.

The final two moments showcase the ability of the model to tackle the international risk sharing puzzle. As expected, the model generates very high correlation between the two pricing kernels; not only is the correlation high on average (0.88), but it is also high across simulations, with the 95% confidence interval being $[0.72, 0.97]$. On the other hand, consumption growth correlation is kept at moderate levels (it is 0.50 on average, and the confidence interval is $[0.20, 0.66]$), with the empirical value (0.33) being somewhat lower than the simulated one, but comfortably within the 95% simulation confidence interval. Figure 1 presents the empirical probability density functions of the domestic and foreign surplus consumption ratio $S = \frac{1}{G}$ and $S^* = \frac{1}{G^*}$, respectively, (Panel (a)) and their ratio $\frac{S}{S^*} = \frac{G^*}{G}$ (Panel (b)): the latter is almost symmetrically distributed around 1, with almost all the probability mass falling in the interval $[0.5, 1.5]$. In fact, great disparities between the two countries' risk aversion ratios are rare: a significant amount of mass is accumulated tightly around 1.

5.4.2 International trade and the real exchange rate

Panel B of Table 3 presents moments related to the real exchange and international trade. First of all, while the model generates (log) real exchange rate changes that are much more volatile than fundamentals (endowment and consumption growth rates), but much less volatile than the pricing kernel, it overshoots empirical real exchange rate change volatility by about 50%. Hence, under the chosen calibration values, the model generates a degree of international risk sharing that is lower than what the data imply. As will be discussed in the next section, this is a not serious shortcoming: even a slight decrease of the domestic home bias parameter can considerably increase international risk sharing and substantially reduce exchange rate change volatility.

On the other hand, the disparity between the simulated and the empirical values for (log) terms of trade change volatility and the correlation between (log) terms of trade movements and (log) real exchange rate movements highlight one of the limitations of the model. Specifically, terms of trade changes are much more volatile in the model than in the data and, while the correlation between terms of trade and real exchange rate changes is perfect in the model, it is far from perfect in the data. Taken together, those two facts imply that the terms of trade are not the only driver of real exchange rates; in reality, a part of the endowment is not tradable, so the existence of non-tradables in both countries affects the properties of the real exchange rate. Nonetheless, the poor performance of the model with respect those two moments may be exaggerated by the fact that the empirical data used to calculate the terms of trade do not match the model definition. Specifically, the terms of trade are calculated as the price of total UK exports over the price of total US exports, when the true definition, according to the model, would be the price of UK exports to the US over the price of US exports to the UK. Since there are no data that would enable the construction of the latter variable, the adopted measure of the terms of trade is imperfect.

Regarding international trade flows, simulated openness ratios (with openness defined as the ratio of the sum of imports and exports over endowment) almost exactly match their empirical values. Furthermore, the net exports ratio for both countries is pro-cyclical in both simulated and empirical data.[30] The last moment of Panel B shows the ability of the model to resolve the Backus and Smith (1993) exchange rate disconnect puzzle: the simulation-generated correlation of consumption growth rate differentials and real exchange rate changes is not only very far from 1 (at 0.14), but also in line with the data.

5.4.3 Asset prices and returns

Panel C of Table 3 evaluates the ability of the model to match asset returns and the relationships between asset prices and the real exchange rate. Regarding the two countries' equity price-dividend ratios, the model produces plausible values for their unconditional mean and generates a cross-country price-dividend ratio correlation that is close to the empirical value. Furthermore,

[30] To be consistent with the international macroeconomics literature, this moment is calculated using the cyclical component of the time-series for both the net exports ratio and the log endowment. The cyclical component for the relevant theoretical and empirical time-series is calculated by applying the Hodrick-Prescott (1997) filter, with sensitivity parameter 1600, on the original series.

the model matches equity excess returns and risk-free rates in both first and second moments, with the exception that simulated excess return volatility is counterfactually high, leading to low model Sharpe ratios. The model also matches empirical excess equity return and risk-free rate cross-country correlations, with a small upward bias: the simulated values are 0.83 and 0.65, with the empirical values being 0.69 and 0.47, respectively.

The rest of the moments focus on the relation between equity prices and the real exchange rate. The model comes close to replicating the zero correlation between real US equity excess returns and US/UK log real exchange rate changes, and it also captures the negative sign - but not the magnitude - of the correlation between UK excess returns and the log real exchange rate changes. Regarding the relationship between the level of the two (log) price-dividend ratios and the level of the (log) real exchange rate, it is clear that, under the model null, those moments are very close to being uninformative: for both countries, the 95% confidence interval is very high.

In short, although the model exhibits considerable ability in capturing most of the asset pricing-related moments, it generates more cross-country correlation between risk-free rates and equity excess returns and more cross-asset correlation between equities and exchange rates than what is found in the data. This should not be very surprising: it is unlikely that two shocks are able to capture all the economic uncertainty in the US and the UK. Another important point is that although the simulated moments correspond to the total wealth portfolio for each country, the actual moments are based on Datastream equity market indices. Since consumption is not equivalent to dividends (and, thus, the total wealth portfolio is not equal to the market portfolio), there is not a perfect mapping between the model and the data in that respect.

5.4.4 Sensitivity with respect to the home bias parameters

The values chosen for the two home bias parameters a and a^*, although clearly motivated by the data, may be extreme, especially regarding the domestic home bias parameter a. To examine the sensitivity of the model results to the home bias parameters, I fix $a^* = 8.2(1-a)$, so as to capture the relative openness (and size) of the two economies, and perform the same simulation exercise as before with $a = 0.95 + 0.005j$, $j = \{0, ..., 10\}$. The results are presented in Figure 2; the horizontal axis measures the value of a and the vertical axis the value of the moment of interest.

First, note that a slight perturbation of the home bias parameter a from 0.99 to 0.98 is sufficient to considerably improve the risk sharing properties of the model:[31] the cross-country pricing kernel correlation increases to 0.96, with the trade-off being that the consumption growth rate correlation increases to 0.59; that is higher than its empirical value, but still much lower

[31] Of course, note that a^* changes much more: specifically, it increases from about 0.08 to about 0.16 in order for relative openness to remain constant.

than 1. The correlation of consumption growth rate differentials with real exchange rate changes, which is the object of the Backus-Smith puzzle, does not change almost at all. The sharp increase in cross-country risk sharing is reflected in the volatility of the real exchange rate, which drops sharply; notice that, for $a = 0.98$, it is slightly lower than the empirical value. The small decrease of domestic home bias also pushes the international correlations of asset prices and returns higher: recall that for the benchmark case of $a = 0.99$, the model cross-country correlation was higher than the empirical correlation for price-dividend ratios, but lower for the risk-free rates and excess equity returns. Furthermore, increased risk sharing appears to weaken the links between asset prices and exchange rates, both in levels and growth rates (returns), bringing the simulated data moments closer to their empirical counterparts. It is clear that a small reduction of the domestic home bias parameter from 0.99 to 0.98 leads to a non-trivial improvement of the model fit with the data.

In contrast to the sometimes sharp changes in unconditional moments when a is reduced from 0.99 to 0.98, further reductions to the value of a do not change much any of the moments of interest. In general, it appears that whether an economy is open or closed has a first order effect on risk sharing and asset price correlations, whereas the degree of openness appears to be a second-order issue. It is, thus, strongly hinted that an economy that trades minimally with the outside world cannot be approximated by a closed economy. The United States, in particular, is relatively closed; however, the fact that it trades, however little, with the rest of the world has a substantial effect on its ability to share risk. That result casts significant doubt to the ability of closed economy asset pricing models to describe economic behavior in economies that exhibit even a minimal amount of international trade.

5.4.5 Robustness with respect to the endowment specification

The assumption that the two countries' (log) endowments are cointegrated is necessary for the (log) real exchange rate to be stationary, so it has a clear economic motivation. However, as discussed in the Appendix (section A.4), an econometric study of the properties of the two endowment processes does not lead to a conclusive answer on whether they are cointegrated. For that reason, in this section I examine the robustness of the simulation results when the cointegration assumption is relaxed. Specifically, assume that endowments satisfy

$$d \log \widetilde{X}_t = \mu^X dt + \sigma^X dB_t^X \tag{40}$$

and

$$d \log \widetilde{Y}_t = \mu^Y dt + \sigma^Y dB_t^Y \tag{41}$$

In that case, $\log z_t$ is a unit root process:

$$d \log z_t = \left(\mu^Y - \mu^X\right) dt + \sigma^z dB_t^z \tag{42}$$

and, consequently, the (log) real exchange rate is non-stationary.

The 5 endowment parameters (μ^X, μ^Y, σ^X, σ^Y and ρ^{XY}) are calibrated by exactly identified GMM estimation as in the stationary case; the estimated parameter values are given in Table 4. The preference parameters are calibrated to have the same values as in the previous section. In results not reported here, after repeating the simulation exercise, the simulated moments are almost exactly identical to the moments calculated in the stationary case, indicating that the model results are robust to endowment specifications that allow for a non-stationary real exchange rate.[32]

6 Conclusion

This paper shows that a two-good, two-country endowment model that incorporates consumption home bias in preferences and external habit formation is able to match several key risk sharing, international trade and asset pricing moments and resolve significant international finance puzzles, including the Brandt, Cochrane and Santa-Clara (2006) international risk sharing puzzle and the Backus and Smith (1993) exchange rate disconnect puzzle. Furthermore, the model shows that, in open economies, foreign preferences and economic conditions can have a significant effect on domestic asset prices and returns. The increasing volume of international transactions in the last few years implies that the asset pricing effects generated by international trade tend to increase with time and cannot be ignored anymore, even for large economies like the US Hence, one the future goals of asset pricing should be to enrich our understanding of the links between asset prices, exchange rates and international risk sharing that characterize open economies.

The model proposed in this paper is quite successful, and, interestingly, that success is accomplished in a stylized environment that exhibits both complete financial markets and frictionless international trade in goods and assets. Naturally, the model is unable to completely describe the economic dynamics of the US and the UK economy. An obvious reason for those shortcomings is that some of the features of the model are unrealistic. Richer models, incorporating frictions in the international trade in goods and assets or incomplete financial markets, may provide a more accurate description of economic reality; this task is left to future research.

[32] The results are not reported in the interests of space; they are available from the author.

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A Appendix

A.1 Solution to the planner's problem

The first order conditions of the planner's problem are:

$$\lambda e^{-\rho t}\pi(\omega,t)a\frac{G_t}{X_t}=\Theta_t$$

$$\lambda e^{-\rho t}\pi(\omega,t)(1-a)\frac{G_t}{Y_t}=\Theta_t^*$$

$$\lambda^* e^{-\rho t}\pi(\omega,t)a^*\frac{G_t^*}{X_t^*}=\Theta_t$$

$$\lambda^* e^{-\rho t}\pi(\omega,t)(1-a^*)\frac{G_t^*}{Y_t^*}=\Theta_t^*$$

where Θ_t and Θ_t^* are the Lagrange multipliers associated with the market clearing condition for the domestic and foreign good, respectively, and $\pi(\omega,t)$ is the P measure probability that state ω occurs at time t.

We now adopt the following notation: $MU_t^X = e^{-\rho t}a\frac{G_t}{X_t}$ is the domestic agent discounted marginal utility of the domestic good and $MU_t^Y = e^{-\rho t}(1-a)\frac{G_t}{Y_t}$ is the domestic agent discounted marginal utility of the foreign good. Similarly for the foreign agent, domestic good marginal utility is $MU_t^{X^*} = e^{-\rho t}a^*\frac{G_t^*}{X_t^*}$ and foreign good marginal utility is $MU_t^{Y^*} = e^{-\rho t}(1-a^*)\frac{G_t^*}{Y_t^*}$. Note that $MU_t^{X^*} = \frac{\lambda}{\lambda^*}MU_t^X$ and $MU_t^{Y^*} = \frac{\lambda}{\lambda^*}MU_t^Y$, i.e. the foreign agent discounted marginal utility for each good is proportional to the respective domestic agent marginal utility. This results from the absence of frictions in international trade for goods: the two agents are able to equalize their marginal utility growth for each of the two goods. Furthermore, the ratio of marginal utilities for the two goods is the same for both countries: $\frac{MU_t^Y}{MU_t^X} = \frac{MU_t^{Y^*}}{MU_t^{X^*}} = \frac{Q_t^*}{Q_t} = Q_t^*$, so the law of one price holds. Denote $\Xi_t = \lambda MU_t^X = \lambda^* MU_t^{X^*}$ and $\Xi_t^* = \lambda MU_t^Y = \lambda^* MU_t^{Y^*}$, so that each agent's marginal utility is proportional to Ξ_t and Ξ_t^*, for the respective good. It follows that $\Xi_t^* = \Xi_t Q_t^*$. Further, $\frac{\Xi_t}{\Xi_0}$ and $\frac{\Xi_t^*}{\Xi_0^*}$ are the state-price deflator processes for the domestic and foreign good, respectively

Using the FOCs, along with the two market clearing conditions, we get the sharing rules (12) and (13). Furthermore:

$$\Xi_t = e^{-\rho t}\left(a\lambda G_t + a^*\lambda^* G_t^*\right)\frac{1}{\widetilde{X}_t} \tag{43}$$

$$\Xi_t^* = e^{-\rho t}\left(\left(1-a\right)\lambda G_t + \left(1-a^*\right)\lambda^* G_t^*\right)\frac{1}{\widetilde{Y}_t} \tag{44}$$

Then, the terms of trade Q_t^* are

$$Q_t^* = \frac{\Xi_t^*}{\Xi_t} = \frac{(1-a)\lambda + (1-a^*)\lambda^* \left(\frac{G_t^*}{G_t}\right)}{a\lambda + a^*\lambda^* \left(\frac{G_t^*}{G_t}\right)} \frac{\widetilde{X}_t}{\widetilde{Y}_t}$$

The domestic agent marginal utility of consumption is $\Lambda_t = e^{-\rho t}\frac{G_t}{C_t}$ and the foreign agent marginal utility of consumption is $\Lambda_t^* = e^{-\rho t}\frac{G_t^*}{C_t^*}$, with $\frac{\Lambda_t}{\Lambda_0}$ and $\frac{\Lambda_t^*}{\Lambda_0^*}$ being the state-price deflator processes for domestic and foreign consumption, respectively. It can easily be shown that

$$\Lambda_t = \frac{1}{\lambda}\left(\frac{\Xi_t}{a}\right)^a \left(\frac{\Xi_t^*}{1-a}\right)^{1-a} \tag{45}$$

$$\Lambda_t^* = \frac{1}{\lambda^*}\left(\frac{\Xi_t}{a^*}\right)^{a^*} \left(\frac{\Xi_t^*}{1-a^*}\right)^{1-a^*} \tag{46}$$

so, using (10), we have

$$E_t = \frac{\lambda^* \Lambda_t^*}{\lambda \Lambda_t}$$

which generates the condition

$$d\log\Lambda_t^* - d\log\Lambda_t = d\log E_t$$

A.2 Pricing kernels

There are four goods in the world economy (the domestic and the foreign good and the domestic and the foreign consumption basket, each of which is a composite good), so we define 4 marginal utility processes: Ξ_t (marginal utility of the domestic good), Ξ_t^* (marginal utility of the foreign good), Λ_t (marginal utility of the domestic consumption basket) and Λ_t^* (marginal utility of the foreign consumption basket). Thus, we can define 4 distinct market price of risk processes, each for one of the 4 aforementioned goods. Formally, the market price of domestic good risk is the bivariate process $\boldsymbol{\eta}_t$ such that

$$\frac{d\Xi_t}{\Xi_t} = -r_t^f dt - \boldsymbol{\eta}_t' \mathbf{dB}_t \tag{47}$$

where r_t^f is the continuously compounded domestic good risk-free rate, i.e. the instantaneous return, in terms of the domestic good, of a locally riskless asset. This asset, called *domestic bond*, has price (in units of the domestic good) D_t and price process $dD_t = r_t^f D_t dt$.

Similarly, the market price of foreign good risk is the bivariate process $\boldsymbol{\eta}_t^*$ such that

$$\frac{d\Xi_t^*}{\Xi_t^*} = -r_t^{f*} dt - \boldsymbol{\eta}_t^{*\prime} \mathbf{dB}_t \tag{48}$$

where $r_t^{f*}$is the foreign good risk-free rate. The locally riskless (in terms of the foreign good) asset is called the *foreign bond;* its price (in units of the foreign good) is D_t^* and its price process is $dD_t^* = r_t^{f*} D_t^* dt$.

We can now define the equivalent terms for the two consumption baskets, C and C^*. The market price of domestic (foreign) consumption risk is the bivariate process $\boldsymbol{\eta}_t^C$ $(\boldsymbol{\eta}_t^{C^*})$ such that

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t^C dt - \boldsymbol{\eta}_t^C \mathbf{dB}_t$$

and

$$\frac{d\Lambda_t^*}{\Lambda_t^*} = -r_t^{C^*} dt - \boldsymbol{\eta}_t^{C^*\prime} \mathbf{dB}_t$$

where, r_t^C and $r_t^{C^*}$ are, respectively, the domestic and the foreign consumption risk-free rate. In other words, r_t^C (r_t^{C*}) is the return of the domestic (foreign) *consumption bond*, an asset that is locally riskless in terms of the domestic (foreign) consumption basket.

It should be noted that, in the case of complete home bias ($a = 1$ and $a^* = 0$), $C = X$ and $C^* = Y$, i.e. the domestic (foreign) consumption basket coincides with the domestic (foreign) good. In that case, $\Xi = \Lambda$ and $\Xi^* = \Lambda^*$, so they have equal market prices ($\boldsymbol{\eta}_t = \boldsymbol{\eta}_t^C$ and $\boldsymbol{\eta}_t^* = \boldsymbol{\eta}_t^{C^*}$) and risk-free rates ($r_t^f = r_t^C$ and $r_t^{f*} = r_t^{C^*}$). This is the case of the standard one-good asset pricing paradigm, in which the consumption good and the endowment good coincide. If home bias is not complete, then the endowment good and the consumption good are not identical. In that case, we will call risk-free rate r_t^f the return of the asset that is riskless in terms of the domestic *good* (not domestic *consumption*); similarly for the foreign risk-free rate r_t^{f*}.

The aforementioned terms are connected. An application of Itô's lemma to (45) and (46) results in:

$$\boldsymbol{\eta}_t^C = a\boldsymbol{\eta}_t + (1-a)\boldsymbol{\eta}_t^*$$

$$\boldsymbol{\eta}_t^{C^*} = a^*\boldsymbol{\eta}_t + (1-a^*)\boldsymbol{\eta}_t^*$$

so the market price of consumption risk (domestic and foreign) is a weighted average (with home biased weights) of the two goods' market price of risk. This makes sense: each country's consumption basket is nothing but a home biased basket of the two goods.

Furthermore, using (45) and (46), we get:

$$r_t^C = ar_t^f + (1-a)r_t^{f*} + \frac{1}{2}a(1-a)\left((\boldsymbol{\eta}_t^* - \boldsymbol{\eta}_t)' \boldsymbol{\Sigma} (\boldsymbol{\eta}_t^* - \boldsymbol{\eta}_t)\right)$$

$$r_t^{C^*} = a^* r_t^f + (1-a^*)r_t^{f*} + \frac{1}{2}a^*(1-a^*)\left((\boldsymbol{\eta}_t^* - \boldsymbol{\eta}_t)' \boldsymbol{\Sigma} (\boldsymbol{\eta}_t^* - \boldsymbol{\eta}_t)\right)$$

The two consumption risk-free rates are home biased weighted averages of the two risk free rates, adjusted by a Jensen inequality term.

A.3 Endowment parameter calibration

To calibrate the 7 endowment-related parameters (μ, θ, ψ, $\bar{z}$, σ^X, σ^Y and ρ^{XY}), I consider the discretized versions of (37) and (38) :

$$\begin{bmatrix} \Delta \log \widetilde{Y}_{t+1} \\ \Delta \log \widetilde{X}_{t+1} \end{bmatrix} = \begin{bmatrix} \mu + (1-\psi)\theta \log \bar{z} \\ \mu - \psi\theta \log \bar{z} \end{bmatrix} + \begin{bmatrix} -(1-\psi)\theta \\ \psi\theta \end{bmatrix} \log z_t + \begin{bmatrix} u^Y_{t+1} \\ u^X_{t+1} \end{bmatrix} \tag{49}$$

with

$$\begin{bmatrix} u^Y_{t+1} \\ u^X_{t+1} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \left(\sigma^Y\right)^2 & \rho^{XY}\sigma^X\sigma^Y \\ \rho^{XY}\sigma^X\sigma^Y & \left(\sigma^X\right)^2 \end{bmatrix} \right) \tag{50}$$

Then, the following 7 moments:

$$\begin{aligned} E\left(u^Y_{t+1}\right) &= 0 \\ E(u^Y_{t+1}\log z_t) &= 0 \\ E(u^X_{t+1}) &= 0 \\ E(u^X_{t+1}\log z_t) &= 0 \\ E\left(\left(u^Y_{t+1}\right)^2\right) - \left(\sigma^Y\right)^2 &= 0 \\ E\left(\left(u^X_{t+1}\right)^2\right) - \left(\sigma^X\right)^2 &= 0 \\ E\left(u^X_{t+1}u^Y_{t+1}\right) - \rho^{XY}\sigma^X\sigma^Y &= 0 \end{aligned} \tag{51}$$

constitute a system of 7 equations and 7 parameters which is estimated by (exactly identified) GMM. The first four moments are the OLS moments for the two processes, while the last 3 moments identify the covariance matrix of the two endowment shocks. The spectral density matrix is Newey-West with 5 lags. The estimation results are presented in Table 1.

A.4 Empirical examination of endowment cointegration

Consider the following specification of the two endowment processes:

$$d\log \widetilde{X}_t = \left[\mu - \psi\left(\delta_0 + \delta_1 \log z_t\right)\right]dt + \sigma^X dB^X_t$$

and

$$d\log \widetilde{Y}_t = \left[\mu + (1-\psi)\left(\delta_0 + \delta_1 \log z_t\right)\right]dt + \sigma^Y dB^Y_t$$

Then, the process for $\log z_t$ is

$$d \log z_t = (\delta_0 + \delta_1 \log z_t)\, dt + \sigma^z dB_t^z$$

It can easily be shown that the above specification yields the stationary case described by (38), (39) and (37) when $\delta_1 < 0$ and the non-stationary case of (40), (41) and (42) when $\delta_1 = 0$ (with constants appropriately renamed).

Discretizing, we can write:

$$\left[\begin{array}{c} \Delta \log \widetilde{Y}_{t+1} \\ \Delta \log \widetilde{X}_{t+1} \end{array} \right] = \left[\begin{array}{c} \mu + (1-\psi)\,\delta_0 \\ \mu - \psi\delta_0 \end{array} \right] + \left[\begin{array}{c} (1-\psi)\,\delta_1 \\ -\psi\delta_1 \end{array} \right] \log z_t + \left[\begin{array}{c} u_{t+1}^Y \\ u_{t+1}^X \end{array} \right]$$

with

$$\left[\begin{array}{c} u_{t+1}^Y \\ u_{t+1}^X \end{array} \right] \sim N\left(\left[\begin{array}{c} 0 \\ 0 \end{array} \right], \left[\begin{array}{cc} \left(\sigma^Y\right)^2 & \rho^{XY}\sigma^X\sigma^Y \\ \rho^{XY}\sigma^X\sigma^Y & \left(\sigma^X\right)^2 \end{array} \right] \right)$$

This a system of balanced regressions, in the sense that the regressor and the dependent variable are of the same order of integration, only if $\log z$ is stationary.[33] In that case, the system above is an error-correction system. To determine whether $\log z$ is a stationary variable, I use the Augmented Dickey-Fuller (ADF) unit root test, when $k = 0, 1, ..., 4$ lagged differences are included in the regression. In results not reported here, the null of $\log z$ being a unit root process cannot be rejected for any conventional levels of significance, for all k.

An alternative procedure is to examine whether $\log \widetilde{Y}$ and $\log \widetilde{X}$ are cointegrated. Note that we can write:

$$\left[\begin{array}{c} \Delta \log \widetilde{Y}_{t+1} \\ \Delta \log \widetilde{X}_{t+1} \end{array} \right] = \left[\begin{array}{c} \mu \\ \mu \end{array} \right] + \left[\begin{array}{c} 1-\psi \\ -\psi \end{array} \right] \left(\delta_0 + \delta_1 \left[\begin{array}{cc} 1 & -1 \end{array} \right] \left[\begin{array}{c} \log \widetilde{Y}_t \\ \log \widetilde{X}_t \end{array} \right] \right) + \left[\begin{array}{c} u_{t+1}^Y \\ u_{t+1}^X \end{array} \right]$$

This is a Vector Error Correction Model (VECM), with the cointegration vector imposed to be $\left[\begin{array}{cc} 1 & -1 \end{array} \right]$. Without imposing this restriction, the system can be written:

$$\left[\begin{array}{c} \Delta \log \widetilde{Y}_{t+1} \\ \Delta \log \widetilde{X}_{t+1} \end{array} \right] = \left[\begin{array}{c} \mu \\ \mu \end{array} \right] + \left[\begin{array}{c} 1-\psi \\ -\psi \end{array} \right] \left(\delta_0 + \delta_1 \left[\begin{array}{cc} 1 & \gamma \end{array} \right] \left[\begin{array}{c} \log \widetilde{Y}_t \\ \log \widetilde{X}_t \end{array} \right] \right) + \left[\begin{array}{c} u_{t+1}^Y \\ u_{t+1}^X \end{array} \right]$$

for $\gamma \in \mathbf{R}$.

[33] In results not reported here, the null of unit root cannot be rejected by the Augmented Dickey-Fuller (ADF) test for neither $\log \widetilde{X}$ nor $\log \widetilde{Y}$, for any conventional level of significance. Furthermore, the ADF test comfortably rejects the unit root null for both $\Delta \log \widetilde{X}$ and $\Delta \log \widetilde{Y}$. Therefore, both $\log \widetilde{X}$ and $\log \widetilde{Y}$ are unit root processes and there is scope for examining whether there exists a cointegrating vector.

I consider the following more general VECM:

$$\begin{bmatrix} \Delta \log \widetilde{Y}_{t+1} \\ \Delta \log \widetilde{X}_{t+1} \end{bmatrix} = A + \Gamma_0 \left(\begin{bmatrix} 1 & \gamma & \gamma_0 \end{bmatrix} \begin{bmatrix} \log \widetilde{Y}_t \\ \log \widetilde{X}_t \\ 1 \end{bmatrix} \right) + \sum_{j=1}^{k-1} \Gamma_j \begin{bmatrix} \Delta \log \widetilde{Y}_{t-j+1} \\ \Delta \log \widetilde{X}_{t-j+1} \end{bmatrix} + \begin{bmatrix} u^Y_{t+1} \\ u^X_{t+1} \end{bmatrix} \tag{52}$$

For $k = 1, ..., 4$, I estimate the number of cointegrating relationships using the Maximum Eigenvalue (λ-max) and the Trace test statistics, as suggested by Johansen (1988, 1991). For each r, the null hypothesis of both the Maximum Eigenvalue and the Trace tests is that there exist exactly r cointegrating relationships; the alternative hypothesis of the former test is that there are $r+1$ cointegrating relationships, whereas the alternative of the latter test is that there are 2 cointegrating relations. The results are presented in Table A1. For all lags, the two test statistics indicate that the null of $r = 0$ cannot be rejected in favor of either $r = 1$ or $r = 2$ for any conventional significance level. However, the two tests also indicate that the null of $r = 1$ cannot be rejected in favor of $r = 2$.

Our empirical results appear to point against the existence of cointegration between the two (log) endowment processes. However, given the properties of the sample under consideration (slightly more than 30 years of quarterly observations) and the economic arguments in favor of real exchange rate stationarity, a definite answer regarding the existence and the properties of a cointegrating vector for the two endowment processes is elusive in this sample.

A.5 Proofs

Proof of Proposition 1

Let

$$\frac{dC_t}{C_t} - E_t\left(\frac{dC_t}{C_t}\right) = \sigma_t^{CX} dB_t^X + \sigma_t^{CY} dB_t^Y \tag{53}$$

and

$$\frac{dC_t^*}{C_t^*} - E_t\left(\frac{dC_t^*}{C_t^*}\right) = \sigma_t^{C^*X} dB_t^X + \sigma_t^{C^*Y} dB_t^Y \tag{54}$$

Using (6) and (8) and applying Itô's lemma, the process of the ratio $\frac{G_t^*}{G_t}$ is:

$$\frac{d\left(\frac{G_t^*}{G_t}\right)}{\left(\frac{G_t^*}{G_t}\right)} = drift + s_t^X dB_t^X + s_t^Y dB_t^Y$$

where

$$
\begin{aligned}
s_t^X &= \delta\left(\frac{G_t - l}{G_t}\right)\sigma_t^{CX} - \delta\left(\frac{G_t^* - l}{G_t^*}\right)\sigma_t^{C^*X} \\
s_t^Y &= \delta\left(\frac{G_t - l}{G_t}\right)\sigma_t^{CY} - \delta\left(\frac{G_t^* - l}{G_t^*}\right)\sigma_t^{C^*Y}
\end{aligned}
$$

Applying Itô's lemma to (14) and (15), we get, respectively:

$$
d\log C_t = drift + a\sigma^X dB_t^X + (1-a)\sigma^Y dB_t^Y - k_t\left(s_t^X dB_t^X + s_t^Y dB_t^Y\right)
$$

$$
d\log C_t^* = drift + a^*\sigma^X dB_t^X + (1-a^*)\sigma^Y dB_t^Y + k_t^*\left(s_t^X dB_t^X + s_t^Y dB_t^Y\right)
$$

where we have used the following definitions for k_t and k_t^*:

$$
k_t \equiv \lambda^* \frac{a(1-a)\lambda + a^*(1-a^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)}{\left(a\lambda + a^*\lambda^*\left(\frac{G_t^*}{G_t}\right)\right)\left((1-a)\lambda + (1-a^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)\right)}\left(\frac{G_t^*}{G_t}\right) \tag{55}
$$

$$
k_t^* \equiv \lambda \frac{a(1-a)\lambda + a^*(1-a^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)}{\left(a\lambda + a^*\lambda^*\left(\frac{G_t^*}{G_t}\right)\right)\left((1-a)\lambda + (1-a^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)\right)} \tag{56}
$$

Regarding k_t, $\lim_{\frac{G_t^*}{G_t}\to 0} k_t = 0$ and $\lim_{\frac{G_t^*}{G_t}\to\infty} k_t = 1$. Further, it can be shown that, for the empirically relevant case $0 < a^* < a < 1$, k_t is globally increasing in $\frac{G_t^*}{G_t}$, so it is bounded in $(0,1)$. Similarly, $\lim_{\frac{G_t^*}{G_t}\to 0} k_t^* = 1$ and $\lim_{\frac{G_t^*}{G_t}\to\infty} k_t^* = 0$. For $0 < a^* < a < 1$, k_t^* is globally decreasing in $\frac{G_t^*}{G_t}$, so it is also bounded in $(0,1)$.

On the other hand, applying Itô's lemma to (53) and (54), we get

$$
d\log C_t = drift + \sigma_t^{CX} dB_t^X + \sigma_t^{CY} dB_t^Y
$$

$$
d\log C_t^* = drift + \sigma_t^{C^*X} dB_t^X + \sigma_t^{C^*Y} dB_t^Y
$$

Matching diffusions, we get the following system of equations:

$$\begin{aligned}
\sigma_t^{CX} &= -k_t\left[\delta\left(\frac{G_t-l}{G_t}\right)\sigma_t^{CX}-\delta\left(\frac{G_t^*-l}{G_t^*}\right)\sigma_t^{C^*X}\right]+a\sigma^X\\
\sigma_t^{CY} &= -k_t\left[\delta\left(\frac{G_t-l}{G_t}\right)\sigma_t^{CY}-\delta\left(\frac{G_t^*-l}{G_t^*}\right)\sigma_t^{C^*Y}\right]+(1-a)\sigma^Y\\
\sigma_t^{C^*X} &= k_t^*\left[\delta\left(\frac{G_t-l}{G_t}\right)\sigma_t^{CX}-\delta\left(\frac{G_t^*-l}{G_t^*}\right)\sigma_t^{C^*X}\right]+a^*\sigma^X\\
\sigma_t^{C^*Y} &= k_t^*\left[\delta\left(\frac{G_t-l}{G_t}\right)\sigma_t^{CY}-\delta\left(\frac{G_t^*-l}{G_t^*}\right)\sigma_t^{C^*Y}\right]+(1-a^*)\sigma^Y
\end{aligned}$$

the solution of which is

$$\left[\begin{array}{c}\sigma_t^{CX}\\ \sigma_t^{C^*X}\end{array}\right]=\frac{1}{D_t^c}\left[\begin{array}{c}a+(ak_t^*+a^*k_t)\,\delta\left(\frac{G_t^*-l}{G_t^*}\right)\\ a^*+(ak_t^*+a^*k_t)\,\delta\left(\frac{G_t-l}{G_t}\right)\end{array}\right]\sigma^X$$

and

$$\left[\begin{array}{c}\sigma_t^{CY}\\ \sigma_t^{C^*Y}\end{array}\right]=\frac{1}{D_t^c}\left[\begin{array}{c}(1-a)+((1-a)k_t^*+(1-a^*)k_t)\,\delta\left(\frac{G_t^*-l}{G_t^*}\right)\\ (1-a^*)+((1-a)k_t^*+(1-a^*)k_t)\,\delta\left(\frac{G_t-l}{G_t}\right)\end{array}\right]\sigma^Y$$

where

$$D_t^c=1+k_t\delta\left(\frac{G_t-l}{G_t}\right)+k_t^*\delta\left(\frac{G_t^*-l}{G_t^*}\right) \tag{57}$$

Proof of Proposition 3

As mentioned in the main text, time t domestic country wealth is the sum of its appropriately discounted future consumption flows. To calculate domestic wealth in units of the domestic good, we convert all future good flows in units of the domestic good and discount using the domestic state-price deflator. Thus

$$W_t=E_t\left[\int_t^\infty\frac{\Xi_s}{\Xi_t}\left(X_s+Q_s^*Y_s\right)ds\right]$$

Similarly, to calculate foreign wealth in units of the foreign good we use:

$$W_t^*=E_t\left[\int_t^\infty\frac{\Xi_s^*}{\Xi_t^*}\left(\frac{X_s^*}{Q_s^*}+Y_s^*\right)ds\right]$$

Using the solutions for Ξ_t, Ξ_t^*, X_t, Y_t, X_t^*, Y_t^* and Q_t^*, we can calculate the wealth of the two countries, given in (26) and (27).

On the other hand the time t value of the domestic country endowment, in units of the domestic good, is $E_t\left[\int_t^\infty\frac{\Xi_s}{\Xi_t}\widetilde{X}_sds\right]$. After some algebra:

$$E_t\left[\int_t^\infty\frac{\Xi_s}{\Xi_t}\widetilde{X}_sds\right]=\frac{(a\lambda+a^*\lambda^*)k\bar{G}+\rho\left(a\lambda G_t+a^*\lambda^*G_t^*\right)}{\rho\left(\rho+k\right)\left(a\lambda G_t+a^*\lambda^*G_t^*\right)}\widetilde{X}_t \tag{58}$$

To calculate λ, we use the fact that, at $t = 0$, the wealth of each country equals the value (in units of the numeraire) of each country's endowment. Therefore, for the domestic country, it holds that

$$W_0 = E_0 \left[\int_0^\infty \frac{\Xi_t}{\Xi_0} \widetilde{X}_t dt \right]$$

Setting $t = 0$, we can get closed-form expressions for W_0 and $E_0 \left[\int_0^\infty \frac{\Xi_t}{\Xi_0} \widetilde{X}_t dt \right]$ from (26) and (58), respectively. Equating the two expressions, as above, and using the normalization $\lambda + \lambda^* = 1$, we derive the expression for λ given in Proposition 3.

Finally, to derive (28), consider that, after substituting the expressions for W_t, W_t^* and Q_t^*, we get

$$\frac{W_t}{W_t + W_t^* Q_t^*} = \frac{\left(\rho G_t + k\bar{G}\right)\lambda}{\left(\rho G_t + k\bar{G}\right)\lambda + \left(\rho G_t^* + k\bar{G}\right)\lambda^*}$$

Setting $t = 0$ and substituting the expressions for λ and λ^* in Proposition 3, we get (28).

Proof of Proposition 4

From (47) and (48) we get

$$r_t^f = -\frac{1}{dt} E_t \left(\frac{d\Xi_t}{\Xi_t} \right)$$

$$r_t^{f^*} = -\frac{1}{dt} E_t \left(\frac{d\Xi_t^*}{\Xi_t^*} \right)$$

so, applying Itô's lemma to (43) and (44) to derive the SDEs that Ξ_t and Ξ_t^* solve and then taking conditional expectations, we arrive at (29) and (30).

Proof of Proposition 5

The price of the domestic total wealth portfolio is (in units of the domestic good):

$$V_t = E_t \left[\int_t^\infty \frac{\Xi_s}{\Xi_t} \widetilde{X}_s ds \right]$$

and, similarly, the price of the foreign total wealth portfolio is (in units of the foreign good):

$$V_t^* = E_t \left[\int_t^\infty \frac{\Xi_s^*}{\Xi_t^*} \widetilde{Y}_s ds \right]$$

Using the expressions for Ξ_t, Ξ_t^*, $\widetilde{X}_t$ and $\widetilde{Y}_s$, we get, after some algebra, (31) and (32).

Proof of Proposition 6

We can define the diffusion processes of the domestic and the foreign total wealth portfolio excess return, $\boldsymbol{\sigma}_t^R$ and $\boldsymbol{\sigma}_t^{R^*}$, respectively, as the bivariate processes such that

$$dR_t^e = \frac{dV_t}{V_t} + \frac{\widetilde{X}_t}{V_t} dt - r_t^f dt = drift + \boldsymbol{\sigma}_t^{R\prime} \mathbf{dB}_t$$

$$dR_t^{e*} = \frac{dV_t^*}{V_t^*} + \frac{\widetilde{Y}_t}{V_t^*}dt - r_t^{f*}dt = drift + \boldsymbol{\sigma}_t^{R^*\prime}\mathbf{dB}_t$$

From (31), we can use Itô's lemma to obtain the SDE satisfied by V_t; then, it is easy to see that the diffusion process of the domestic total wealth portfolio is given by (34).

The domestic good market price of risk $\boldsymbol{\eta}_t$ is defined in (47) and, applying Itô's lemma to (43), we get that:

$$\frac{d\Xi_t}{\Xi_t} = drift - \left(\sigma^X \mathbf{e}_1 + \left(\omega_t \boldsymbol{\sigma}_t^G + (1-\omega_t)\boldsymbol{\sigma}_t^{G^*}\right)\right)\mathbf{dB}_t$$

so, equating diffusions, we get (33).

Similarly, applying Itô's lemma to get the SDE for V_t^*, we get that the diffusion process of the foreign total wealth portfolio is given by (36). An application of Itô's lemma to (44) gives

$$\frac{d\Xi_t^*}{\Xi_t^*} = drift - \left(\sigma^Y \mathbf{e}_2 + \left(\omega_t^* \boldsymbol{\sigma}_t^G + (1-\omega_t^*)\boldsymbol{\sigma}_t^{G^*}\right)\right)\mathbf{dB}_t$$

so the foreign good market price of risk, defined in (48), is given by (35).

Therefore, the excess return of the domestic total wealth portfolio is

$$dR_t^e = \mu_t^R dt + \boldsymbol{\sigma}_t^{R\prime}\mathbf{dB}_t$$

where μ_t^R is the domestic total wealth portfolio conditional risk premium, calculated as

$$\mu_t^R = -\frac{1}{dt}E_t\left(dR_t^e \frac{d\Xi_t}{\Xi_t}\right) = \boldsymbol{\eta}_t^\prime \boldsymbol{\Sigma}\boldsymbol{\sigma}_t^R$$

Similarly, the excess return of the foreign total wealth portfolio is

$$dR_t^{e*} = \mu_t^{R^*} dt + \boldsymbol{\sigma}_t^{R^*\prime}\mathbf{dB}_t$$

where $\mu_t^{R^*}$ is the foreign total wealth portfolio conditional risk premium, given by

$$\mu_t^{R^*} = -\frac{1}{dt}E_t\left(dR_t^{e*} \frac{d\Xi_t^*}{\Xi_t^*}\right) = \boldsymbol{\eta}_t^{*\prime} \boldsymbol{\Sigma}\boldsymbol{\sigma}_t^{R^*}$$

Table 1

Endowment calibration

Parameter	Estimate
μ	0.0037
	(0.0006)
θ	0.0520
	(0.0204)
ψ	0.0195
	(0.1436)
$\log \bar{z}$	-1.0348
	(0.0243)
σ^X	0.0074
	(0.0007)
σ^Y	0.0195
	(0.0018)
ρ^{XY}	0.1553
	(0.0786)

The endowment parameters in (37) and (38) are estimated by exactly identified GMM, with moment conditions given in (51). The spectral density matrix is Newey-West with 5 lags. Standard errors in parentheses. Note: the parameters are not annualized.

Table 2

Calibration Parameters (annualized)

Endowment parameter	Symbol	Value
Steady-state of endowment ratio z	$\bar{z}$	1
Speed of z mean reversion	θ	0.192
Endowment growth rate	μ	0.015
Domestic contribution to endowment adjustment	ψ	0.02
Domestic endowment growth volatility	σ^X	0.015
Foreign endowment growth volatility	σ^Y	0.039
Endowment growth correlation	ρ^{XY}	0.155
Preference parameter		Value
Domestic preference for the domestic good	α	0.990
Foreign preference for the domestic good	α^*	0.082
Subjective rate of time preference	ρ	0.04
Speed of G mean reversion	k	0.12
G sensitivity to consumption growth shocks	δ	79.39
Lower bound of G	l	20
Steady-state value of G	$\bar{G}$	34

Calibration parameters. All parameters are annualized. The endowment parameters in Table 2 are the annualized counterparts of the parameters in Table 1, with the exception of the steady-state endowment ratio $\bar{z}$, which is normalized to 1.

Table 3

Simulation Results

A. Endowment and consumption

Moment	*Model*		*Data*	
	US	*UK*	*US*	*UK*
Relative endowment value $\left(\frac{\widetilde{X}}{\widetilde{Y}Q^*}\right)$ mean	8.23 [6.97, 9.47]		8.29	
Endowment growth mean	1.50% [1.00%, 2.02%]	1.50% [0.79%, 2.19%]	1.48%	2.01%
Endowment growth st. dev.	1.50% [1.31%, 1.69%]	3.95% [3.48%, 4.44%]	1.49%	3.98%
Endowment growth correlation	0.15 [−0.02, 0.32]		0.15	
Endowment growth autocorrelation	−0.01 [−0.18, 0.16]	−0.02 [−0.19, 0.15]	−0.01	−0.29
Consumption growth mean	1.50% [1.00%, 2.01%]	1.50% [0.88%, 2.10%]	1.80%	2.08%
Consumption growth st. dev.	1.46% [1.25%, 1.68%]	2.92% [1.95%, 4.29%]	1.19%	2.05%
Consumption growth autocorrelation	−0.01 [−0.18, 0.16]	−0.02 [−0.24, 0.19]	0.33	−0.18
Consumption growth correlation	0.50 [0.20, 0.66]		0.33	
Log pricing kernel correlation	0.88 [0.72, 0.97]		–	

B. International trade and the real exchange rate

Moment	*Model*		*Data*	
	US	*UK*	*US*	*UK*
Log real exchange rate change st. dev	16.27% [12.27%, 23.19%]		10.26%	
Log terms of trade change st. dev.	17.92% [13.51%, 25.54%]		3.75%	
$corr(\log Q^*, \log E)$	1.00 [1.00, 1.00]		0.23	
Openness mean	0.02 [0.02, 0.02]	0.16 [0.15, 0.18]	0.02	0.17
Correlation of NX with endowment	0.24 [−0.16, 0.60]	0.63 [0.24, 0.88]	0.33	0.28
$corr(\Delta e_{t+1}, \Delta c^*_{t+1} - \Delta c_{t+1})$	0.14 [−0.49, 0.52]		0.05	

Table 3 (cont.)

Simulation Results

C. Asset prices and returns

Moment	*Model*		*Data*	
	US	*UK*	*US*	*UK*
Log pricing kernel st. dev.	34.50% [16.81%, 69.53%]	37.09% [19.46%, 73.10%]	-	-
Equity P/D mean	32.10 [27.09, 34.88]	31.90 [26.38, 35.14]	40.73	25.13
Equity P/D correlation	0.79 [0.45, 0.97]		0.94	
Equity excess return mean	6.74% [2.88%, 10.01%]	8.46% [3.66%, 12.70%]	7.68%	7.92%
Equity excess return st. dev.	22.03% [13.95%, 30.90%]	25.15% [17.38%, 33.78%]	15.53%	16.16%
Equity excess return correlation	0.83 [0.69, 0.92]		0.69	
Sharpe ratio	0.32 [0.11, 0.54]	0.35 [0.13, 0.58]	0.49	0.49
Risk-free rate mean	0.46% [−1.25%, 3.54%]	0.14% [−2.66%, 4.09%]	1.80%	2.71%
Risk-free rate st. dev.	1.67% [0.80%, 3.03%]	2.01% [1.07%, 3.26%]	1.40%	2.52%
Risk-free rate correlation	0.65 [0.18, 0.93]		0.47	
$corr(\Delta \log E, R^e)$	0.10 [−0.29, 0.45]	−0.40 [−0.72, 0.02]	0.00	−0.13
$corr(\log E, \log(P/D))$	0.13 [−0.58, 0.74]	−0.39 [−0.88, 0.42]	0.32	0.20

A comparison of simulated and empirical moments. To calculate the former, I simulate 10,000 sample paths of the model economy, with each path consisting of 170 quarterly observations. The system is initialized at $z_1 = \bar{z}$, $G_1 = G_1^* = \bar{G}$ and $\widetilde{X}_1 = 1$. Of the 170 observations, the first 40 (10 years) are discarded to reduce the dependence on initial conditions Thus, each sample path consists of 130 observations, as many as available in the dataset. For each of the moments of interest, Table 3 presents the sample average across the 10,000 simulations, as well as the 2.5% and 97.5% percentiles across simulations (in brackets). The empirical moments are calculated by using data from 1975:Q1 to 2007:Q2, for a total of 130 quarterly observations.

Table 4

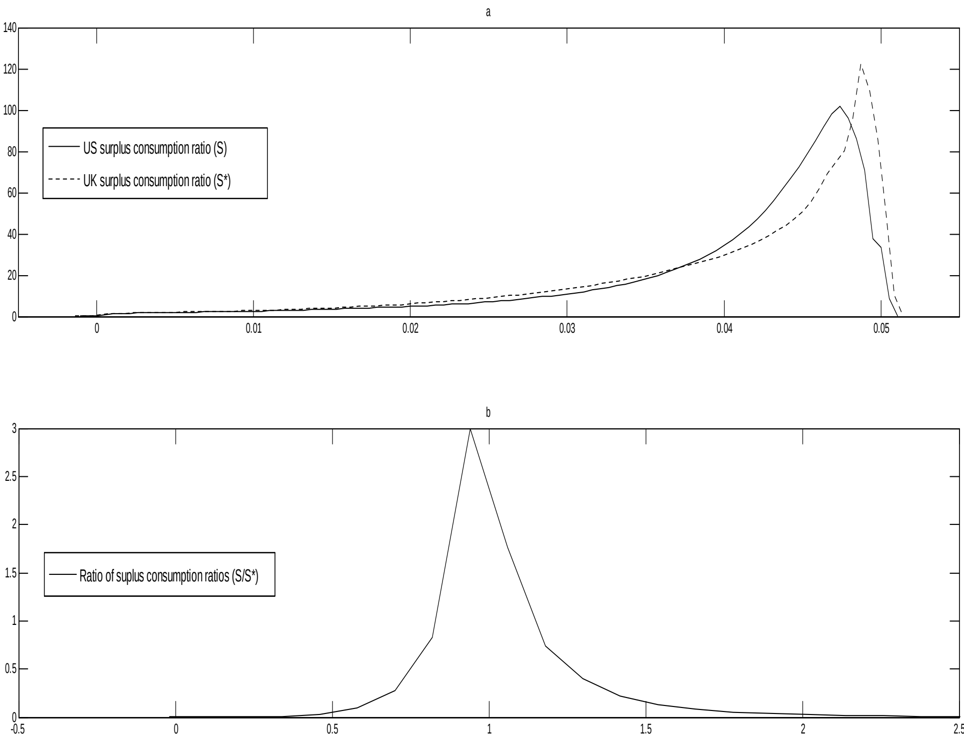
Endowment calibration: non-stationary case

Parameter	Estimate
μ^X	0.0037
	(0.0007)
μ^Y	0.0050
	(0.0011)
σ^X	0.0074
	(0.0007)
σ^Y	0.0198
	(0.0018)
ρ^{XY}	0.1512
	(0.0780)

The parameters are estimated by exactly identified GMM, using as moment conditions the first, third, fifth, sixth and seventh conditions in (51). The spectral density matrix is Newey-West with 5 lags. Standard errors in parentheses. Note: the parameters are not annualized.

Figure 1

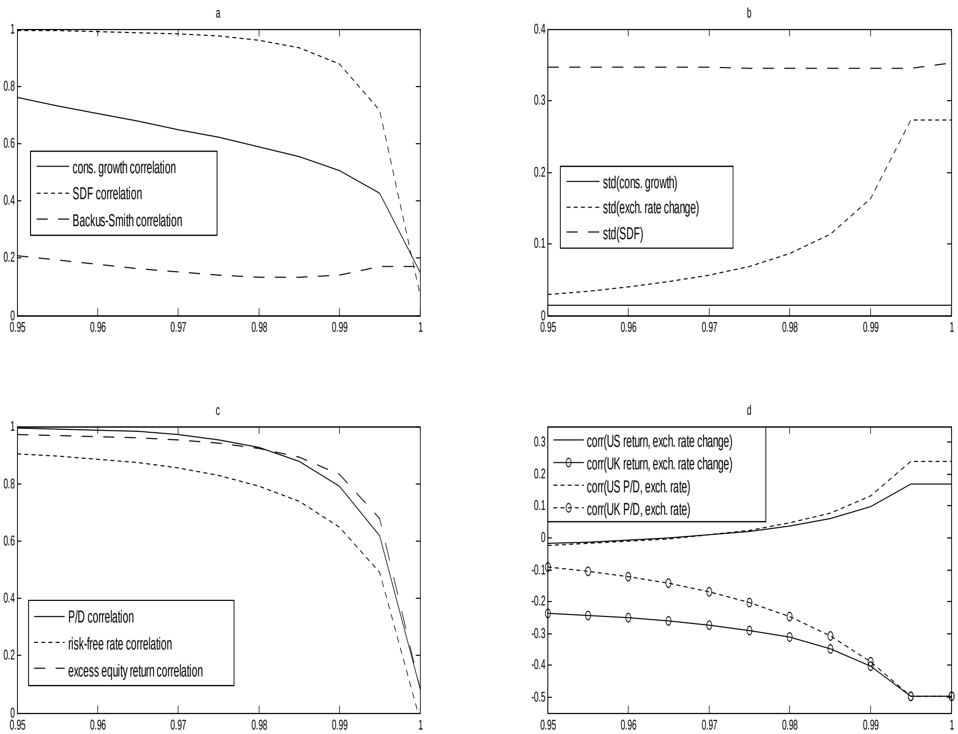
Empirical probability density functions of surplus consumption ratios



Empirical probability density functions (PDFs). Panel (a) presents the empirical PDF of the domestic and foreign surplus consumption ratio $S = \frac{1}{G}$ and $S^{*} = \frac{1}{G^{*}}$ (for the United States and United Kingdom, respectively). Panel (b) presents the empirical PDF of the ratio $\frac{S}{S^{*}} = \frac{G^{*}}{G}$. The empirical estimate is calculated by using a normal kernel.

Figure 2

Simulated moments for different values of a



Simulated moments. I examine the sensitivity of model results to the home bias parameters by fixing $a^{*}= 8.2(1-a)$, so as to capture the relative openness of the two economies, and vary the domestic home bias parameter a: $a = 0.95 + 0.005j,\ j = \{0, ..., 10\}$. For each of the values of a, I simulate 10,000 sample paths of the model economy and, for each of the moments of interest, I calculate the sample average across the 10,000 simulations. The horizontal axis measures the value of a and the vertical axis the value of the moment of interest. Panel (a) presents the correlation between the domestic and the foreign consumption growth rate, the correlation between the domestic and the foreign pricing kernel and the Backus and Smith (1993) puzzle correlation $corr(\Delta e_{t+1}, \Delta c^{*}_{t+1} - \Delta c_{t+1})$. Panel (b) presents the standard deviation of: the domestic consumption growth rate, the domestic pricing kernel and log real exchange rate changes. Panel (c) presents the correlation between domestic and foreign risk-free rates and between the excess returns and price-dividend ratios of the two countries' total wealth portfolios. Panel (d) presents, for each of the two countries, the correlation: (i) between the excess return of the total wealth portfolio and the change in the log real exchange rate, and (ii) between the log price-dividend ratio of the total wealth portfolio and the log real exchange rate.

Table A1

Cointegration test for $\log \widetilde{X}$ and $\log \widetilde{Y}$

r	λ-max	$p-value$	Trace	$p-value$
		$k=1$		
0	9.10	0.28	9.12	0.35
1	0.02	0.88	0.02	0.88
		$k=2$		
0	7.79	0.40	7.90	0.48
1	0.10	0.75	0.10	0.75
		$k=3$		
0	8.98	0.29	9.02	0.36
1	0.03	0.86	0.03	0.86
		$k=4$		
0	9.91	0.22	9.91	0.29
1	0.00	0.95	0.00	0.95

Results of Johansen cointegration tests for $\log \widetilde{X}$ and $\log \widetilde{Y}$; the econometric specification is given by (52). For the null hypothesis of $r = 0$ and $r = 1$ cointegrated vectors, the Maximum Eigenvalue (λ-max) and Trace test statistics are presented in the second and fourth column, respectively; the p-values for each test are presented in the third and fifth columns.